QUESTION 1) The circular beam shown in the figure is subjected to a fluctuating force. The direction and the magnitude of the force varies from 2F in positive y direction to F in negative y direction, in each cycle. The beam is cold rolled AISI 1030 Steel and the semi-circular groove and the fillet are machined surfaces. The groove has a radius of \( r_1 = 4 \text{ mm} \) and the fillet has radius of \( r_2 = 3 \text{ mm} \). Find the maximum permissible limit of force F for an infinite life of operation, if the reliability and the factor of safety are 0.90 and 2 respectively. Use Modified Goodman Diagram in the failure analysis (\( S_{ut} = 520 \text{ MPa}, S_y = 440 \text{ MPa} \)).

Critical sections of the beam are section-I, where the beam has a groove, and section-II, where the cross-section changes from 30 mm to 40 mm. Actually, the moment created by the forces acting on the beam is maximum at the section where the beam is fixed to the wall. However, the beam is fixed to the wall with large radius fillet (which reduces the stress concentration at this point) and the ratio of the moment at this section to the moment at section-I is:

\[
\frac{M}{M_I} = \frac{170*F}{150*F} = 1.13
\]

which is small compared with the stress concentration factor at section-I.

During the application of force 2F, lowermost points of the beam are in tension. Since tensile stresses are more critical in failure analysis, points A and B will be checked (Type of Fatigue Loading; Fluctuating).

Cold drawn AISI 1030 Steel: \( S_{ut} = 520 \text{ MPa} \quad S_y = 440 \text{ MPa} \)

At point A

\[
C_{load} = 1.0
\]

\[
C_{surface} = A*(S_{ut})^b; A = 4.51, b = -0.265; \quad \text{see pp. 351}
\]

\[
C_{size} = 1.189*(d^{-0.097}) = 0.86 \quad \text{see pp. 348}
\]

\[
C_{size} = 0.85 \quad (\text{for} \ 8 \leq d \leq 250 \text{ mm})
\]
reliability factor: $C_{\text{relab}} = 0.897$ (0.9 reliability, pp. 353)

temperature factor: $C_{\text{temp}} = 1$ (not mentioned)

In order to find fatigue strength reduction factor we have to find fatigue stress concentration factor, $K_f$.

$$K_f = 1 + q(K_r - 1)$$

$q = 0.82$ (r = 4 mm = 0.16 in, Sut = 520 MPa = 75 ksi; A = 0.86 via linear interpolation; see Table 6-6, pp. 362)

$K_r = 1.8$ (r/d = 4/32 = 0.125, D/d = 40/32 = 1.25, figure E-5, pp. 996; A = 0.944, b = -0.31)

$K_f = 1 + 0.82 \cdot (1.8 - 1) = 1.7$ (Assume that $K_f|\Sigma_{\text{max}}| < S_y$: hence $K_f = K_f^*$; see Eq. 6-17, pp. 385)

Assume that $K_m = K_f = 1.7$ (you can later check that if this assumption is correct)

$$S_e^* = 0.5 \cdot S_{\text{ut}} = 0.5 \cdot 520 = 260 \text{ MPa}$$

$$S_e = C_{\text{load}} \cdot C_{\text{surf}} \cdot C_{\text{size}} \cdot C_{\text{temp}} \cdot C_{\text{rel}} \cdot S_e^* = (1.0) \cdot (0.86) \cdot (0.85) \cdot (1) \cdot (0.897) \cdot 260 = 171 \text{ MPa}$$

The bending stress at point A is:

$$\sigma = \frac{M \cdot c}{I}$$

$M_{\text{max}} = 150 \cdot F \text{ (N.mm)}$ $M_{\text{min}} = -150 \cdot F \text{ (N.mm)}$

$d = 40 - 2.4 = 32 \text{ mm}$

$I = \pi \cdot \frac{d^4}{64}$

$c = \frac{d}{2}$

$$\sigma_{\text{min}} = -0.047 \cdot F \text{ MPa}$$

$$\sigma_{\text{max}} = 0.093 \cdot F \text{ MPa}$$

$$\sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}$$

$$\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}$$

$$\sigma_m = (1.7) \cdot 0.023 \cdot F = 0.039 F \text{ MPa}$$

$$\sigma_a = (1.7) \cdot 0.070 \cdot F = 0.119 F \text{ MPa}$$

Using Modified Goodman Diagram:

Find the intersection point of Yield and Goodman lines:

solve:

$$\sigma_a' + \sigma_m' = \sigma_y / n$$

$$\sigma_a' / S_e + \sigma_m' / S_{\text{ut}} = 1 / n; \text{ where } n=2$$

$$\sigma_a' = 20 \text{ MPa}$$

$$\sigma_m' = 200 \text{ MPa}$$

$$\alpha = \tan^{-1} (\sigma_a' / \sigma_m') = 5.7^\circ$$

with the stresses that we have calculated, find the slope of load line:

$$\beta = \tan^{-1} (\sigma_a / \sigma_m) = \tan^{-1} ((1.7 \cdot 0.070) / (1.7 \cdot 0.023)) = 71.6^\circ$$
α<β which means;

The failure in this problem will be fatigue failure, so we will use Goodman line. If α was close to β we should check the stresses with yielding line.

\[
\frac{\sigma_a}{S_v} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \quad \frac{0.119F}{171} + \frac{0.039F}{520} = \frac{1}{2}
\]

which gives \( F = 656.5 \) N

**At point B**

\( G_{load} = 1 \quad C_{surf} = 0.86 \quad C_{reliab} = 1 \) (same as point A)

\( G_{size} = 1.189(24)^{-0.097} = 0.87 \)

(for \( 8 \leq d \leq 250 \))

\( C_{temp} = 1.0 \)

\( q = 0.82 \) (Table 6-6)

\( K_r = 1.61 \)

\( (r/d = 3/30 = 0.1, \ D/D = 40/30 = 1.33, \ A = -0.95, b = -0.23; \ figure \ E-2, \ pp. \ 994) \)

\( K_f = 1 + 0.82(1.61 - 1) = 1.5 \)

\( S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_{v}' = 174 \) MPa

The bending stress at point B is:

\[
\sigma = \frac{M \cdot c}{I}
\]

\( M_{max} = 100 \cdot 2 \cdot F \) (N.mm) \quad \( M_{min} = -100 \cdot F \) (N.mm)

\( d = 30 \text{ mm} \quad I = \frac{\pi \cdot d^4}{64} \quad c = d / 2 \)

\( \sigma_{min} = -0.038 \cdot F \) MPa

\( \sigma_{max} = 0.075 \cdot F \) MPa

\[
\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad \sigma_u = \frac{\sigma_{max} - \sigma_{min}}{2}
\]

\( \sigma_m = 0.019 \cdot F \) MPa \quad \( \sigma_u = 0.057 \cdot F \) MPa

Using Modified Goodman Diagram:

with the stresses that we have calculated, the slope of load line:

\( \beta = \tan^{-1}(\sigma_u / \sigma_m) = 71.6^\circ \)

\( \alpha < \beta \)
again we will use Goodman line: \[ \frac{\sigma_u}{S_c} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \]

\[ \frac{0.086 F}{174} + \frac{0.029 F}{520} = \frac{1}{2} \]

which gives \( F = 935 \) N

So Point A is more critical and \( F_{\text{max}} = 656.5 \) N for infinite operation life.
QUESTION 2) A rotating circular shaft, machined from AISI 1095 Q&T steel, is subjected to a torque that varies from a value of 200 N.m to 400 N.m and to a fluctuating bending moment that varies from a value of -100 N.m to 300 N.m. Also an axial tensile force of 5 kN acts on the element. The torque and the bending moments have their peak values at the same time and their frequencies are same. Find the factor of safety of the element for an infinite operation life assuming the reliability as 0.999. Use Modified Goodman Diagram (Material Properties; $S_{ut} = 1260$ MPa, $S_y = 813$ MPa).

\[
\begin{align*}
    d_1 &= 60 \text{ mm} \quad d_2 = 40 \text{ mm} \quad r = 8 \text{ mm}
\end{align*}
\]

**Solution:**

There is a combined loading in this problem. The load factor and stress concentration factor differ according to the type of loading. Therefore, they will be used as load modifying factors instead of endurance limit modifying factors. They will be used for modifying the alternating components of the stresses (The critical section is at the fillet).

For the material: $S_{ut} = 1260$ MPa $S_y = 813$ MPa

- $C_{surf} = A \times (S_{ut})^b = 0.76$ (Machined surfaces, Table 6-6; $A = 4.51$, $b = -0.265$)
- $C_{relab} = 0.753$ (0.999 reliability, Table 6-4)
- $C_{size} = 1.189 \times d^{-0.097} = 1.189 \times (40^{-0.097}) = 0.83$
- $S_e = (0.76) \times (0.753) \times (C_{temp} = 1) \times (0.83) \times (0.5) \times (1260) = 300$ MPa

Geometric properties:

\[
\begin{align*}
    \frac{r}{d} &= \frac{8}{40} = 0.2 \\
    \frac{D}{d} &= \frac{60}{40} = 1.5
\end{align*}
\]

**Axial Loading**

- $C_{load} = 0.7$ (for axial loading)
- $K_t = 1.55$ (using figure E-1, pp. 994; the formula will return more accurate values!)
- $q = 0.96$ (Sut = 1260 MPa = 0.145*1260 kpsi = 182.7 kpsi; From Table 6-6, we obtain $\sqrt{a} = 0.024$ in; you can also use Fig 6-36)
- $K_t = 1 + q (K_t - 1) = 1.53$
- $k_e = 1 / (C_{load} \times K_t) = 0.93$

Axial load is constant, it will only create mean stress.

\[
\begin{align*}
    \sigma_m &= \frac{F}{A} \\
    F &= 5000 \text{ N} \\
    A &= \pi \times \frac{d_2^2}{4} \\
    \sigma_m &= \frac{3.979}{\text{MPa}}
\end{align*}
\]
**Bending Moment**

- $C_{\text{load}} = 1.0$
- $K_t = 1.4$ (figure E-2)
- $q = 0.96$
- $K_f = 1 + q (K_t - 1) = 1.38$
- $k_e = 1 / K_f = 0.72$

Bending moment is fluctuating. So it will create both mean and alternating stresses.

\[
\sigma = \frac{M \cdot c}{I} = \frac{100000 \cdot mm}{\pi \cdot \frac{d^2}{64}}
\]

\[
M_{\min} = -100000 \, \text{N-mm} \quad M_{\max} = 300000 \, \text{N-mm}
\]

\[
M_m = \frac{M_{\max} + M_{\min}}{2} = 100000 \, \text{N-mm}
\]

\[
M_a = \frac{M_{\max} - M_{\min}}{2} = 200000 \, \text{N-mm}
\]

\[
\sigma_m = \frac{M_m \cdot c}{I} = 15.915 \, \text{MPa}
\]

\[
\sigma_a = \frac{M_a \cdot c}{I} = 31.831 \, \text{MPa}
\]

\[
\sigma_a = \frac{\sigma_a}{C_{\text{load}} \cdot k_e} = 44.2 \, \text{MPa}
\]

**Torsion**

- $C_{\text{load}} = 1$
- $K_t = 1.24$ (using fig E-3)
- $q = 0.96$
- $K_f = 1 + q (K_t - 1) = 1.23$
- $k_e = 1 / K_f = 0.81$

Torsion is fluctuating. So it will create both mean and alternating shear stresses.

\[
\tau = \frac{T \cdot c}{J} = \frac{200000 \cdot mm}{\pi \cdot \frac{d^2}{32}}
\]

\[
T_{\min} = 200000 \, \text{N-mm} \quad T_{\max} = 400000 \, \text{N-mm}
\]

\[
T_m = \frac{T_{\max} + T_{\min}}{2} = 300000 \, \text{N-mm}
\]

\[
T_a = \frac{T_{\max} - T_{\min}}{2} = 100000 \, \text{N-mm}
\]

\[
\tau_m = \frac{T_m \cdot c}{J} = 23.873 \, \text{MPa}
\]

\[
\tau_a = \frac{T_a \cdot c}{J} = 7.958 \, \text{MPa}
\]

\[
\tau_a = \frac{\tau_a}{C_{\text{load}} \cdot k_e} = 9.83 \, \text{MPa}
\]

Total results:

\[
\sigma_m = 15.915 \, \text{MPa} + 3.979 \, \text{MPa} = 19.894 \, \text{MPa} \quad \text{(bending + axial force)}
\]
\[ \sigma_a = 44.2 \text{ MPa} \]
\[ \tau_m = 23.873 \text{ MPa} \]
\[ \tau_a = 9.83 \text{ MPa} \]

According to Von-Misses, 2D Stress State:
\[ \sigma_m = \sqrt{\sigma_m^2 + 3 \cdot \tau_m^2} = \sqrt{(19.894)^2 + 3(23.873)^2} = 45.887 \text{ MPa} \]
\[ \sigma_a = \sqrt{\sigma_a^2 + 3 \cdot \tau_a^2} = \sqrt{(44.20)^2 + 3(9.83)^2} = 47.37 \text{ MPa} \]

slope of the load line: \( \beta = \tan^{-1}(\sigma_a' / \sigma_m') = 46^\circ \)

To find the intersection point of Yield and Goodman lines, solve:
\[ \sigma_a + \sigma_m = \sigma_y \]
\[ \sigma_a / S_e + \sigma_m / S_u = 1 \]
\[ \sigma_a = 139.7 \]
\[ \sigma_m = 673.3 \]
\[ \alpha = \tan^{-1}(\sigma_a / \sigma_m) = 11.7^\circ \]

\( \alpha < \beta \): Failure will be fatigue. Therefore, we will use Goodman line,
\[ \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_u} = \frac{1}{n} \]
which gives \( n = 5.15 \)
QUESTION 3) A machine part, made up of AISI 1060 Annealed Steel is first subjected to a completely reversed stress of magnitude 220 MPa, for 1000 days with 100 cycles/day. Then the fully reversed loading is increased to 250 MPa and it’s applied for 500 days with the same number of cycles per day. Initial Endurance limit of the part is 180 MPa (Material Properties of AISI 1060; $S_{ut} = 625$ MPa, $S_y = 372$ MPa).

After these loadings:

i) What’s the endurance limit of the part?

ii) For how many days it can be used under a fully reversed load of 180 MPa?

Use Manson’s rule in the analysis.

Solution:

$$S_{ut} = 626 \text{ MPa} \quad S_y = 372 \text{ MPa} \quad S_e = 180 \text{ MPa}$$

Total cycles for first loading: $n_1 = 1000 \text{ days} \cdot 100 \text{ cycles/day} = 100000 \text{ cycles}$

Corresponding lifetime ($N_1$) for 220 MPa loading in the initial S-N curve.

$$\log \frac{220 - \log 180}{\log (0.8 \cdot 626) - \log 180} = \frac{6 - \log(N_1)}{6 - 3} \quad N_1 = 258028 \text{ cycles}$$

Remaining life of the part for 220 MPa loading: $N_{r1} = N_1 - n_1 = 258028 - 100000 = 158028 \text{ cycles}$

The material is damaged after this loading. So, the S-N graph must be redrawn such that it will pass through the point $(\log(158028), \log(220))$.

The new endurance limit is:

$$\log \frac{500.8 - \log 220}{\log 500.8 - \log S_{e1}} = \frac{\log 158028 - 3}{6 - 3} \quad S_{e1} = 163 \text{ MPa}$$
Total cycles for second loading: \( n_2 = 500 \text{ days} \cdot 100 \text{ cycles/day} = 50000 \text{ cycles} \)

Corresponding lifetime \( (N_1) \) for 250 MPa loading in the modified S-N curve.

\[
\frac{\log 250 - \log 163}{\log(500.8) - \log 163} = \frac{6 - \log(N_2)}{6 - 3} \rightarrow N_2 = 71951 \text{ cycles}
\]

remaining life of the part for 250 MPa loading: \( N_{r2} = N_2 - n_1 = 71951-50000 = 21951 \text{ cycles} \)

The S-N graph must be modified again such that it will pass through the point \( \log(21951), \log(250) \).

i) The final endurance limit is:

\[
\frac{\log 500.8 - \log 250}{\log 500.8 - \log S_{e2}} = \frac{6 - \log(N_{e2})}{6 - 3} \rightarrow S_{e2} = 106 \text{ MPa}
\]

ii) Remaining lifetime \( N_3 \) for 180 MPa loading in the last modified S-N curve.

\[
\frac{\log 180 - \log 106}{\log(500.8) - \log 106} = \frac{6 - \log(N_3)}{6 - 3} \rightarrow N_3 = 94570 \text{ cycles}
\]

94570 cycles / 100 cycles per day = 945 days