Example Problem: The steel pipe shown in the figure has an inner diameter of 2 cm and outer diameter of 2.15 cm. If it is subjected to the force $F = 80j + 40k$ (N), determine the principal stresses in the pipe at point A and B which are located on the surface of the pipe.
**Solution:**

Hollow tube:

\[ I = \frac{\pi}{4} \left[ \left( \frac{2.15 \times 10^{-2}}{2} \right)^4 - \left( \frac{2.0 \times 10^{-2}}{2} \right)^4 \right] = 2.63 \times 10^{-9} \text{ m}^4 \]

\[ J = \frac{\pi}{2} \left[ \left( \frac{2.15 \times 10^{-2}}{2} \right)^4 - \left( \frac{2.0 \times 10^{-2}}{2} \right)^4 \right] = 5.26 \times 10^{-9} \text{ m}^2 \]

\[ A = \pi \left[ \left( \frac{2.15 \times 10^{-2}}{2} \right)^2 - \left( \frac{2.0 \times 10^{-2}}{2} \right)^2 \right] = 4.88 \times 10^{-5} \text{ m}^2 \]

Free-Body Diagram

- \( M_x = 80 \times 10^{-2} = 8 \text{ Nm} \)
- \( M_y = 40 \times 12^{-2} = 4.8 \text{ Nm} \)
- \( T = 80 \times 12^{-2} = 9.6 \text{ Nm} \)
- \( V_1 = 80 \text{ N} \)
- \( V_2 = 40 \text{ N} \)

**Applied Stresses at Point A:**

**Normal Stress (Axial Tension):**

\[ \sigma_z = \frac{V_2}{A} = \frac{40}{4.88 \times 10^{-5}} = 8.2 \times 10^5 \text{ Pa} \]

(Along + z axis)
Normal Stresses (Axial Tension)

Normal stress due to the bending moment about the x-axis = 0

Normal stress due to the bending moment about the y-axis:

\[ \sigma_z = \frac{M_y r_o}{I} = \frac{(4.8) \times \left(\frac{2.15 \times 10^{-2}}{2}\right)}{2.63 \times 10^{-3}} = 1.96 \times 10^7 \text{ Pa} \text{ (along } -z \text{ axis)} \]
Shear stress due to the bending moment about the x-axis

- Shear force (V1) is acting along the positive y-axis. Hence, the shear stress is on the XY plane and acting in the direction of positive y-axis.
- The shear stress is max along the moment axis. Hence, the shear stress is max at Point A and zero at Point B.
- Check the ratio of wall thickness to outer radius; \(0.075/1.075 = 0.07 < 0.1\)

\[ \tau_{zy} = \frac{2V}{A} = \frac{2 * 80}{4.88 \times 10^{-3}} = 32.8 \times 10^3 \text{ Pa} \]  
(Along the +y axis)

Torsional shear stress:

\[ \tau_{zy} = \frac{Tr}{J} = \frac{9.6 \times (2.15 \times 10^{-3})}{2 \times 5.26 \times 10^{-9}} = 1.96 \times 10^7 \text{ Pa} \]  
(Along the –y axis; see the diagram)
Total Normal Stress:

$$\sigma_z = 8.2 \times 10^5 - 196 \times 10^5 \approx -187 \times 10^5 \text{ Pa}$$

Total Shear Stress:

$$\tau_{zy} = 32.8 \times 10^5 - 196.10^5 \text{ Pa} \approx -163 \times 10^5 \text{ Pa}$$

Principal Stresses at Point A:
The principal stresses can be computed as

$$\sigma_{1,2} = \frac{\sigma_y + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + (\tau_{zy})^2}$$

$$\sigma_{1,2} = \frac{-187 \times 10^5}{2} \pm \sqrt{\left(\frac{-187 \times 10^5}{2}\right)^2 + (-163 \times 10^5)^2}$$

$$\sigma_1 = 94.4 \times 10^5 \text{ Pa}$$

$$\sigma_2 = -281.4 \times 10^5 \text{ Pa}$$

Applied Stresses at Point B:
Normal Stress (Axial Tension):

$$\sigma_z = \frac{V_z}{A} = \frac{40}{4.88 \times 10^{-5}} = 8.2 \times 10^5 \text{ Pa} \quad \text{(along } +z \text{ axis)}$$

Normal stress due to the bending moment about the x-axis:

$$\sigma_z = \frac{M_x r_o}{I} = \frac{(8) \times \left(2.15 \times 10^{-2}\right)}{2.63 \times 10^{-9}} = 3.3 \times 10^7 \text{ Pa} \quad \text{(along } +z \text{ axis)}$$
Normal stresses due to the bending moment about the X-axis

\[ \sigma_z = 0 \]

Normal stress due to the bending moment about the y-axis = 0

Shear stress due to the bending moment about the x-axis = 0

Torsional shear stress:

\[ \tau_{zx} = \frac{Tr}{J} = \frac{9.6 \times (2.15 \times 10^{-2})}{2 \times 5.26 \times 10^{-9}} = 1.96 \times 10^7 \text{ Pa} \] (along the +x axis; see the diagram)
Total Normal Stress:

\[ \sigma_z = 8.2 \times 10^5 + 330 \times 10^5 \approx 338 \times 10^5 \text{ Pa} \]

Total Shear Stress:

\[ \tau_{zx} = 196.10^5 \text{ Pa} \]

**Principal Stresses at Point B:**
The principal stresses can be computed as

\[
\sigma_{1,2} = \frac{\sigma_z + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + (\tau_{zx})^2}
\]

\[
\sigma_{1,2} = \frac{338 \times 10^5}{2} \pm \sqrt{\left(\frac{338 \times 10^5}{2}\right)^2 + (196 \times 10^5)^2}
\]

\[ \sigma_1 = 427.8 \times 10^5 \text{ Pa} \]

\[ \sigma_2 = -89.8 \times 10^5 \text{ Pa} \]