Chapter 13: Springs

Outline

- Spring Functions & Types
- Helical Springs
  - Compression
  - Extension
  - Torsional

The Function(s) of Springs

Most fundamentally: to STORE ENERGY

Many springs can also: push pull twist

Some Review

Some Review

\[ k = \frac{F}{y} \]

Parallel

\[ k_{total} = k_1 + k_2 + k_3 \]

Series

\[ \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \]

Types of Springs

Helical:

- Standard constant rate
- Variable pitch rate
- Conical

Compression

Extension

Torsion

More Springs

Washer Springs:

Beams:

Power springs:
**Helical Compression Springs**

- Number of coils: $N_t$
- Diameter of wire: $d$
- Mean coil diameter: $D$
- Free length: $L_f$
- Pitch: $p$
- Total coils: $N_t$

May also need: $D_o$ and $D_i$

**Length Terminology**

- Free Length: $L_f$
- Assembled Length: $L_a$
- Max Working Load: $L_m$
- Bottomed Out: $L_s$

**End Conditions**

- Plain
- Plain Ground
- Square
- Square Ground

$N_a = \text{Active Coils}$

**Stresses in Helical Springs**

- Spring Index: $C = \frac{D}{d}$
- Typically: $4 \leq C \leq 12$

$\tau_{\text{max}} = K_w \frac{8FD}{nd^2}$, where $K_w = \frac{2C+1}{2C}$

$K_w$ includes both the direct shear factor and the stress concentration factor.

- Under static loading, local yielding eliminates stress concentration, so use $K_s$
- Under dynamic loading, failure happens below $S_y$: use $K_s$ for mean, $K_w$ for alternating

**Curvature Stress**

Inner part of spring is a stress concentration

$K_w$ includes both the direct shear factor and the stress concentration factor.

$\tau_{\text{max}} = K_w \frac{8FD}{nd^2}$, where $K_w = \frac{4C-1}{4C-4} \cdot 0.615 \frac{C}{d}$

- Under static loading, local yielding eliminates stress concentration, so use $K_s$
- Under dynamic loading, failure happens below $S_y$: use $K_s$ for mean, $K_w$ for alternating

**Spring Deflection**

$y \approx \frac{8FD^3N_a}{d^4G}$
Spring Rate

\[ y \approx \frac{8FD^3Na}{d^4G} \]
\[ k = \frac{d^4G}{8D^3Na} \]

Helical Springs

- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Extension
- Torsion

Static Spring Design

- Inherently iterative
  - Some values must be set to calculate stresses, deflections, etc.
- Truly Design
  - There is not one “correct” answer
  - Must synthesize (a little bit) in addition to analyze

Material Properties

- \( S_{ut} \) ultimate tensile strength
  - Figure 13-3
  - Table 13-4 with \( S_{ut} = Ad^b \)
- \( S_{ys} \) torsional yield strength
  - Table 13-6 – a function of \( S_{ut} \) and set

Spring/Material Treatments

- Setting
  - Overstress material in same direction as applied load
    - Increase static load capacity 45-65%
    - Increase energy storage by 100%
    - Use \( K_s \) not \( K_y \) (stress concentration relieved)
- Load Reversal with Springs
- Shot Peening
  - What type of failure would this be most effective against?

What are You Designing?

<table>
<thead>
<tr>
<th>Given</th>
<th>Find</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F, y )</td>
<td>( k )</td>
</tr>
<tr>
<td>( k, y )</td>
<td>( F )</td>
</tr>
</tbody>
</table>

Such that:
- Safety factor \( s > 1 \)
- Spring will not buckle
- Spring will fit in hole, over pin, within vertical space

* - Often can calculate from given
** - Often given/defined
Static Spring Flow Chart

if GIVEN F, y, then find k; if GIVEN k, y, then find F

- \( D, K_p, K_m \)
- material strengths
- \( N_s = \frac{F}{\tau} \)
- for shut spring if possible
- if not, for max working load

if GIVEN \( F, y \), then find \( k \); If GIVEN \( k, y \), then find \( F \)

- \( D, K_s, K_w \)
- material strengths

Three things to know:
- effect of \( d \)
- shortcut to finding \( d \)
- how to check buckling

ITERATE?

Check:
- buckling, \( N_{shut}, D_i, D_o \)

\( N_{shut} = \frac{F_{sys}}{\tau_{shut}} \)

Helical Springs

- Compression
- Nomenclature
- Stress
- Deflection and Spring Constant
- Static Design
- Fatigue Design
- Torsion

Material Properties

- \( S_{utt} \) ultimate shear strength
  - \( S_{utt} = 0.67 S_u \)
- \( S_{utw} \) torsional fatigue strength
  - Table 13-7 -- function of \( S_{utt} \), # of cycles
  - repeated, room temp, 50% reliability, no corrosion
- \( S_{tw} \) torsional endurance limit
  - for steel, \( d < 10 \text{mm} \)
  - see page 816 (45 ksi (310 MPa) if unpeened,
  - =67.5 ksi (465 MPa) if peened)
  - repeated, room temp., 50% reliability, no corrosion

Modified Goodman for Springs

- \( S_{uts}, S_{utw} \) are for torsional strengths, so von Mises not used

\[ S_{fi} = 0.5 \left( \frac{S_{uts} S_{utw}}{S_{uts} - 0.3(S_{uts})} \right) \]
Fatigue Safety Factor

\[ \tau_a \]

\[ N_f = \frac{S_{a}}{\tau_a} \]

Fatigue Spring Design Strategy

if GIVEN F, y, then find k; If GIVEN k, y, then find F

STRESSES

\[ N_y = \frac{f_x(S_{a} - \tau_i)}{f_y(S_{a} - \tau_i) + f_y} \]

DEFLECTION

\[ L = X_{max} - F_{max} \]

ITERATE?

CHECK

Two things to know:
- shortcut to finding d
- how to check frequency

Fatigue Design: Wire Diameter

as before, you can iterate to find d, or you can use an equation derived from relationships that we already know:

\[ d = \left( \frac{NCN \Delta F_{min}}{0.6742} \right) \left( K_{F} \left( N_{F} - 1 \right) - \left( 1.3 \Delta d \right) \left( \frac{N_{F}}{N_{F_{min}}} \right) \right)^{1/2} \]

use Table 13-2 to select standard d near calculated d

Two things to know:
- shortcut to finding d
- how to check frequency

**maintain units (in. or mm) for A, b**

Natural Frequency: Surge

Surge == longitudinal resonance

for fixed/fixed end conditions:

\[ f_s = \frac{1}{2} \sqrt{\frac{K_{F}}{W_{a}}} \] (Hz)

ideally, \( f_s \) will be at least 13\( x \) more than \( f_{s,\text{surge}} \), it should definitely be multiple times bigger

Two things to know:
- shortcut to finding d
- how to check frequency

...see pages 814-815 for more

What are you Designing?

Given

\[ F_{\text{max}}, F_{\text{min}}, \gamma, k, \alpha \]

Find

\[ k, \gamma, C, D', F, N_{\text{shut}}, \text{allowance (\alpha)}, \text{material}^{*} \]

Such that:

Fatigue Safety Factor is > 1
Shut Static Safety Factor is > 1
Spring will not buckle
Spring is well below natural frequency
Spring will fit in hole, over pin, within vertical space

* - often can calculate from Given
** - often given/defined

Review of Design Strategy

ITERATIVE

Find Loading
Select C, d

Find stresses
Determine material properties
Find safety factor

USING d EQUATION

Find Loading
Select C, safety factor

Solve for d, pick standard d
Find stresses
Determine material properties
Check safety factor
Strategy Review
Continued

- Find spring constant, $N_a, N_t$
- Find $F_{SHUT}$ (must find lengths and $y$'s to do this)
- Find static shut shear stress and safety factor

Check Buckling
Check Surge
Check $D_i, D_o$ if pin to fit over, hole to fit in

Consider the Following:

Helical Springs
- Compression
  - Nomenclature
  - Stress
  - Deflection and Spring Constant
  - Static Design
  - Fatigue Design
- Torsion

Torsion Springs

Deflection & Spring Rate

\[
\theta_{rev} = \left( \frac{M}{EI} \right) \frac{\theta}{\theta_{rev}}
\]

\[
\theta_{rev,round} = 10.2 \left( \frac{M}{d^3} \right) \left( \frac{D_N}{d^4} \right)
\]

\[
k = \frac{M}{\theta_{rev}}
\]

Materials

see Tables 13-13 and 13-14, page 850

follow book on $S_{crit} = S_{crit} / 0.577...$ for now

Stresses

1. Static - Compressive is Max \( \sigma = K_b \frac{M_{\text{max}}}{d^3} \) - Inside of Coil

\[
\sigma_{\text{max}} = K_b \frac{32M_{\text{max}}}{\pi d^3}
\]

\[
K_b = \frac{4c^2 - C - 1}{4c(C + 1)}
\]

2. Fatigue - (since fatigue is a tensile stress phenomenon) - Outside of Coil

\[
\sigma_{\text{max}} = K_b \frac{32M_{\text{max}}}{\pi d^3}
\]

\[
\sigma_{\text{min}} = K_b \frac{32M_{\text{min}}}{\pi d^3}
\]

\[
K_b = \frac{4c^2 - C - 1}{4c(C + 1)}
\]
Strategy

Select C, d

• fit over pin (if there is one)
• don’t exceed stresses

Helical Springs

- Compression
- Nomenclature
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- Deflection and Spring Constant
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- Fatigue Design
- Torsion