EXAMPLE

When the system shown in Figure 5–56(a) is subjected to a unit-step input, the system output responds as shown in Figure 5–56(b). Determine the values of $K$ and $T$ from the response curve.

Solution. The maximum overshoot of 25.4% corresponds to $\zeta = 0.4$. From the response curve we have

$\tau_p = 3$

Consequently,

$\tau_p = \frac{\pi}{\omega_n} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = \frac{\pi}{\omega_n \sqrt{1 - 0.4^2}} = 3$

It follows that

$\omega_n = 1.14$

From the block diagram we have

$\frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K}$

from which

$\omega_n = \sqrt{\frac{K}{T}}$, $\quad 2\zeta \omega_n = \frac{1}{T}$

Therefore, the values of $T$ and $K$ are determined as

$T = \frac{1}{2\zeta \omega_n} = \frac{1}{2 \times 0.4 \times 1.14} = 1.09$

$K = \omega_n^2 T = 1.14^2 \times 1.09 = 1.42$

Figure 5–56
(a) Closed-loop system; (b) unit-step response curve.