Pergamon

J. Biomechanics, Vol. 27, No. 12, pp. 1495–1499, 1994

0021–9290/94 $7.00+.00

0021–9290(94)E0020–4

TECHNICAL NOTE

PRESENTING JOINT KINEMATICS OF HUMAN LOCOMOTION USING PHASE PLANE PORTRAITS AND POINCARÉ MAPS*

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Abstract—Additional graphical tools are needed to better visualize the joint kinematics of human locomotion. Standard plots in which the joint displacements are plotted against time or percent gait cycle do not provide sufficient information about the dynamics of the system. In this article, a study based on the two graphical tools of nonlinear dynamics to visualize the steady-state kinematics of human gait is presented. An experimental setup was developed to acquire the necessary data for application of the techniques. Twenty young adults, whose medical histories are free of gait pathology, were tested. Computerized electrogoniometers and foot switches were used to measure the kinematic data of the lower extremities and capture four instants of the gait cycle: heel strike, foot flat, heel off, and toe off. Phase plane portraits of each joint were constructed for the sagittal plane by plotting angular velocity against angular displacement. Poincaré maps were obtained by periodically sampling the joint profiles at toe off and plotting the ith iterate against the $(i+1)$th one. Phase plane portraits are useful in monitoring the variations of joint velocity and position on the same graph in a more compact form. Poincaré maps are effective in differentiating steady gait from transient locomotion.

INTRODUCTION

Typical measurements that are obtained in the contemporary gait laboratory fall into three major categories. The first category consists of kinematic measurements such as joint rotations, cadence, stride length, and walking speed. The second group includes kinetic data consisting mainly of joint resultant loads and power transfers, which are obtained from force plate measurements to compute the ground reaction forces. The third group includes physiological measures such as EMG, ECG, oxygen consumption, and carbon dioxide production (Davis, 1988; Inman *et al.*, 1981; Whittle, 1991).

Experimentally measured joint profiles are often used in the evaluation of gait. This investigation presents an approach that is geared toward developing a set of new techniques to graphically represent kinematic data of human locomotion. The proposed graphical tools are based on the methods of nonlinear dynamics. We use Poincaré maps to identify steady-state locomotion (dynamic equilibrium) of human subjects by using experimentally measured kinematic data. Upon this identification, phase plane portraits are utilized to intuitively visualize the joint kinematics of steady gait. The approach is a direct extension of the methodology that was used to study the stability and global bifurcations of synthesized bipedal machines in Hurmuzlu (1993a, b).

The utility of the proposed techniques was demonstrated by conducting an experimental investigation of the locomotion of 20 normal young adults. We used electrogoniometers as the principal measurement device for acquiring kinematic data because of their availability and their relative ease of use (Isacson *et al.*, 1986).

THEORY

The motion of the simplified three-link biped [Fig. 1(a)] evolves in the six-dimensional state space, $\{\theta_1, \theta_2, \theta_3, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\}$.† In Fig. 1(b), point 1 on the phase plane portrait (a plot of angular position versus angular velocity) is the time instant when the biped is swinging leg B in the forward direction. At point 2, the swing limb contacts the ground surface causing an impulsive change in the velocity (point 3) and a pivot transfer to the contact point of limb B with the ground. Thereafter, the biped continues to its forward motion swinging on leg B. The subsequent contact occurs at point 4, and the cycle of events repeats. The motion that is depicted in Fig. 1(b) is not periodic because points 1 and 5 of the phase trajectory do not coincide. When the motion becomes periodic, the trajectory loop closes on itself (loop 1–2–3–4).

Plotting phase plane portraits of human gait requires the measurement of the angular displacements and computation of the velocities by numerically differentiating the position data. For the human gait, however, the sudden velocity changes of the present rigid body model are not observed. The effect of heel strike in human locomotion is considerably less critical because the impulsive forces are absorbed by the soft tissue.

During ambulation, a particular trajectory descends to the closed orbit with successive locomotion steps [Fig. 2(a)]. The points of the trajectory that coincide with the instant of heel strike are labeled as p_i and the point of the closed orbit at heel strike is labeled as p_e. The Poincaré map of θ_1, at the instant of heel strike, can now be obtained by plotting the values of

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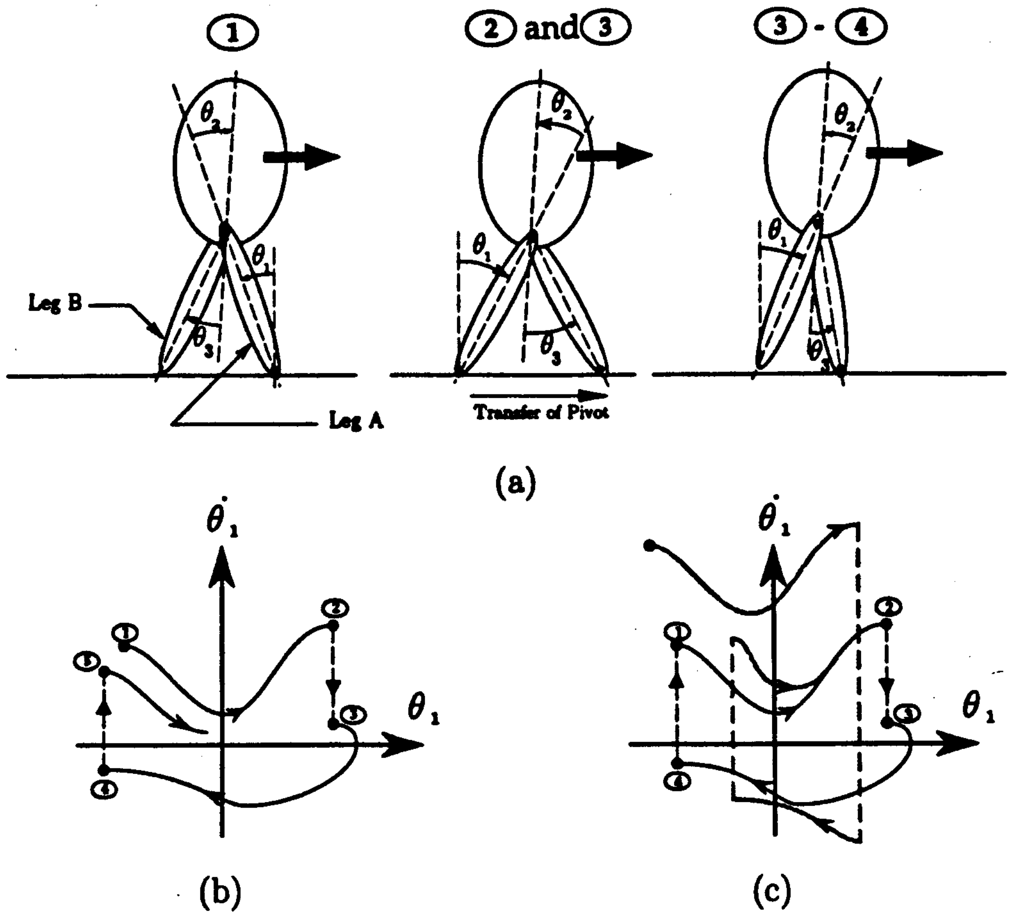
Received in final form 14 *February* 1994.
* Supported by a grant from the Whitaker Foundation.

† Dots denote differentiation with respect to time (e.g. $\dot{\theta}_1$ denotes the angular velocity of joint 1).



(a)

(b)

(c)

Fig. 1. Phase plane portrait and periodic motions of a three-link model.

θ_1 at p_i versus the values at p_{i+1} [Fig. 2(b)]. The point p_e is on the 45° line because when the motion is periodic, θ_1 at heel strike is identical for all successive locomotion steps. In addition, one can observe that points on the map accumulate at p_e as the biped takes successive locomotion steps. The same construction can also be performed for any other kinematic quantitty. One can also use other foot timing events instead of the heel strike.

METHODS

The principal device used for kinematic data collection during the course of the present study was a computerized triaxial electrogoniometer system (Chattex Corp.) A modified version of this system was used to measure the angular joint displacements about three, mutually perpendicular axes at the hip, knee, and the ankle of the subject's dominant side. The following notation was used for joint rotations:

ϕ_1, ϕ_4, ϕ_7 — sagittal plane excursion of ankle, knee, and hip joints, respectively.
ϕ_2, ϕ_5, ϕ_8 — coronal plane excursion of ankle, knee, and hip joints, respectively.
ϕ_3, ϕ_6, ϕ_9 — transverse plane excursion of ankle, knee, and hip joints, respectively.

Foot switches located at under the heel and the forefoot are used to delineate the beginning and the end of stance and swing phases during gait.

Twenty adult subjects free of known gait pathologies (nine males and 11 females) volunteered for the present study. The male subjects ranged in ages from 19 to 36 yr (mean = 26.6 ± 7.0) and weighed between 50 and 85 kg (mean = 68.2 ± 13.3). The female subjects were between 19 and 49 yr (mean = 28.9 ± 10.1) and weighed between 50 and 83 kg (mean = 59.0 ± 9.0).

The subjects were asked to walk over the 20 m walkway 16 times in succession. They were instructed to walk at their most comfortable speed. The joint excursions of each subject were collected by means of an electrogoniometry system and stored in a file via a personal computer. The angular velocities of joints were computed by first performing a second-degree binomial smoothing and then numerically differentiating the position data. The following interpolation scheme was developed to calculate the means and standard deviations of the kinematic variables.

(1) Kinematic data from the toe off to the subsequent toe off of the 12th locomotion step of the 16 walking passes performed by all subjects were extracted. We have selected the 12th step because it generally occurred at the middle of the walkway, where one expects to encounter steady gait.

(2) The time scale of each data set was normalized over the duration of the respective step. Then, interpolation was used to obtain 129 evenly spaced data points spanning over the step duration.

(3) Means and standard deviations of each kinematic variable were computed by processing the kinematic data at the 129 evenly spaced time instances that were obtained in the previous step.

The first return maps (Poincaré maps) of joint coordinates and velocities were obtained by periodically sampling the

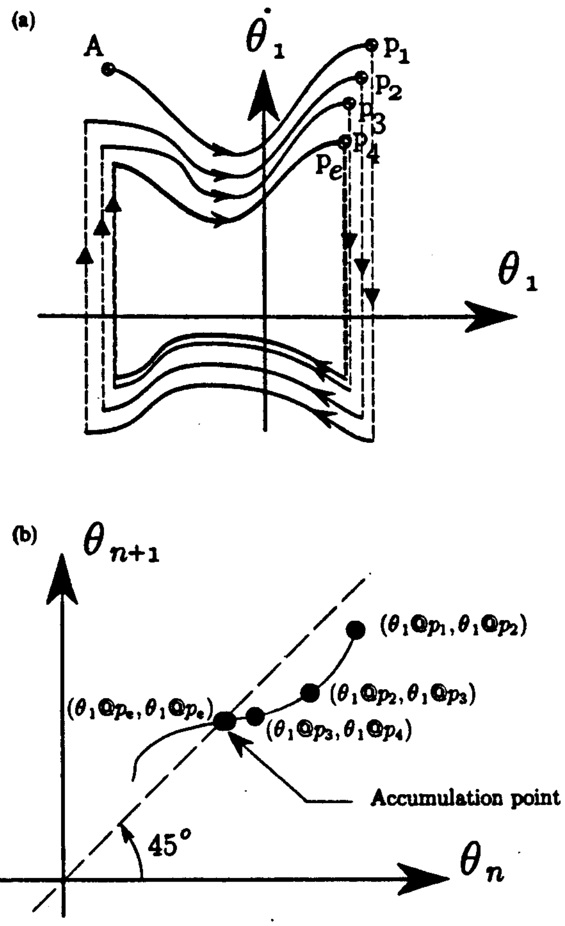


Fig. 2. Equilibrium points and the first return map.

joint kinematics at the instant of toe off (arbitrarily chosen from other detectable foot contact events). For each map, the maximum and minimum values on the horizontal and vertical axes were picked as the actual variation limits of the particular kinematic variable during the entire walking process.

RESULTS

The average of gait parameters for the locomotion of the female and male subjects of the present study were 113 ± 11 and 106 ± 9 steps min^{-1} for cadence and 1.33 ± 0.12 and 1.29 ± 0.17 m s^{-1} for velocity, respectively.

Sagittal phase plots of a typical female subject [Fig. 3(a)] denote that the knee joint attains the highest velocities and undergoes the largest coordinate change during the gait cycle. However, the motion of the knee is mainly during the swing phase. One can also observe that this joint comes to a complete stop and attains its flexion and extension limits in the early and late swing phases. During the stance phase, however, the knee joint is the least active one. As far as the hip joint is concerned, it is the most active joint during the stance phase. We also note that the ankle and the knee joints are stationary at the heel-off instant.

We detected periodic steady state in the locomotion of normal subjects. Periodicity was observed from the accumulation of iterates on the sagittal maps around the fixed points (Fig. 4). The averages of all passes at the Poincaré section (toe off) just before the 12th step and at that step were considered as i and $(i+1)$ dynamic equilibrium values of the respective variables (Table 1). Since the joint profiles were not perfect sinusoidals, the average first return values did not match exactly. Circles that are centered at the mean values were drawn as visual aids to detect the accumulation regions. The radii of the circles were selected as 10% of the diagonal distances of the plots. The circles were sized as such for no specific reasons other than graphical perspective. Although Fig. 4 represents the maps of one subject, centering of the circles with respect to the accumulation points has been observed in the sagittal maps of all subjects.

DISCUSSION

In this article, phase plane portraits and first return maps were introduced as graphical tools to visualize the joint kinematics of human gait. Compared to the customary time displacement plots, phase plane portraits lead to a more direct representation of joint positions and velocities in single plot. First return maps are specialized plots of periodically sampled kinematic data that can be effectively used to distinguish the steady-state gait from transient locomotion.

In establishing the profiles for the normal population, we limited our analysis to the sagittal plane rotations. This was due to the difficulties associated with the alignment of goniometers along the anatomical joint axes in the other two planes. We believe that the proposed tools can be used for other planes of motion, subject to the availability of a data acquisition system which is less susceptible to alignment

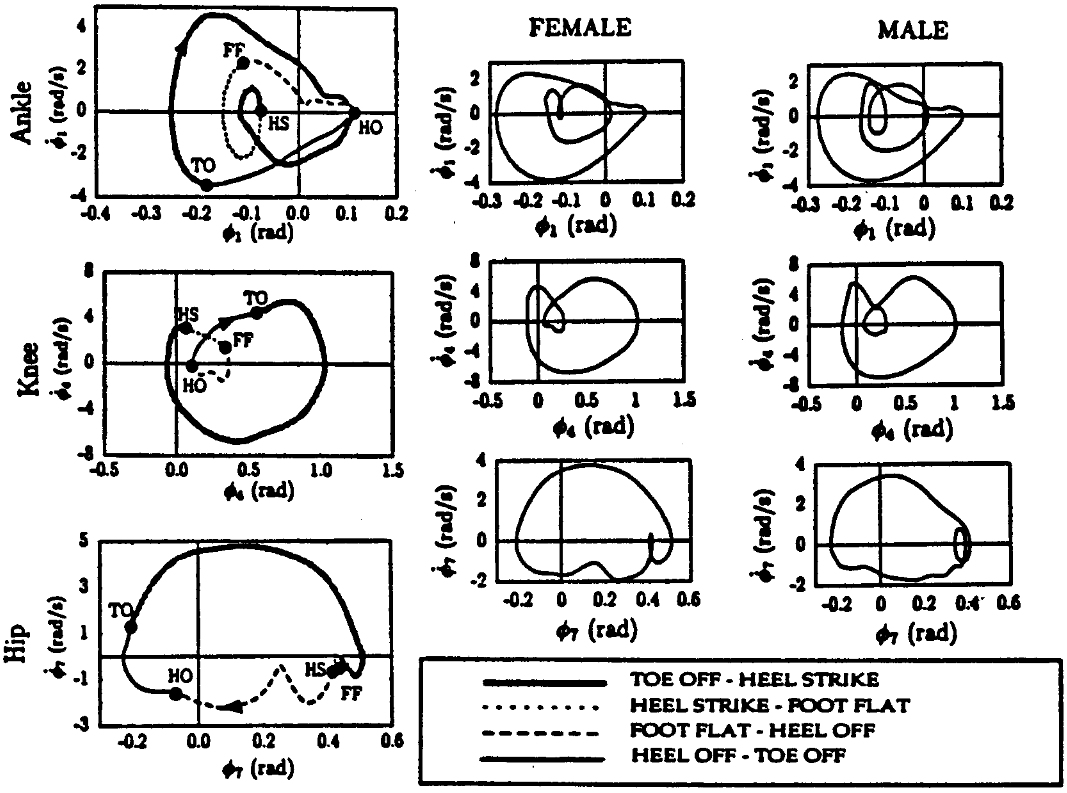


Fig. 3. (a) The detailed phase portraits of a typical female subject (the left column). (b) Phase portraits of healthy female subjects in sagittal plane (the middle column). (c) Phase portraits of healthy male subjects in sagittal plane (the right column). Phase plane portraits combine position and velocity data on a single plot. Steady state joint, velocities can be correlated directly with positions by eliminating the time variable.

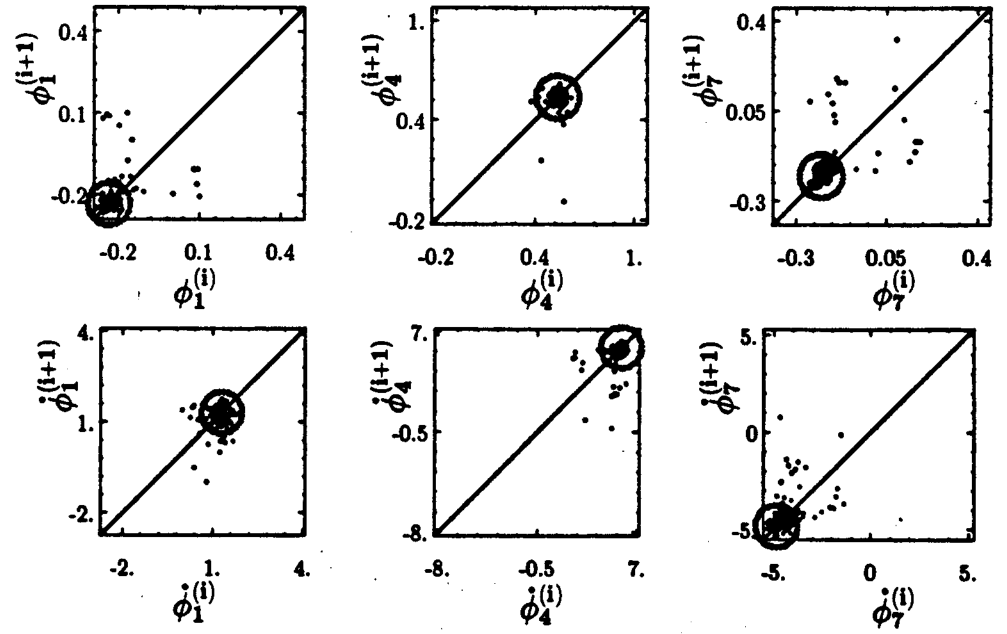


Fig. 4. Sagittal maps of a normal male subject. First return maps are graphical tools that facilitate in distinguishing between transient and steady state locomotion. Steady state locomotion can be observed from clustering of points inside the shown circles.

Table 1. Equilibrium values of sagittal joint rotations and velocities of normal subjects at four instances of locomotion

Event	Group	$\phi_1^{(i)}$	$\phi_1^{(i+1)}$	$\phi_4^{(i)}$	$\phi_4^{(i+1)}$	$\phi_7^{(i)}$	$\phi_7^{(i+1)}$
Heel strike	Males	−0.05 ± 0.11	−0.05 ± 0.12	−0.02 ± 0.08	−0.02 ± 0.09	0.38 ± 0.09	0.38 ± 0.09
	Females	−0.15 ± 0.10	−0.14 ± 0.10	−0.09 ± 0.15	−0.08 ± 0.15	0.37 ± 0.10	0.37 ± 0.10
Foot flat	Males	−0.04 ± 0.11	−0.04 ± 0.12	−0.22 ± 0.10	−0.21 ± 0.10	0.30 ± 0.19	0.30 ± 0.19
	Females	−0.07 ± 0.09	−0.07 ± 0.10	−0.20 ± 0.11	−0.20 ± 0.11	0.31 ± 0.13	0.31 ± 0.13
Heel off	Males	0.11 ± 0.11	0.11 ± 0.12	0.13 ± 0.12	0.13 ± 0.12	0.02 ± 0.17	0.03 ± 0.17
	Females	0.08 ± 0.09	0.09 ± 0.08	0.12 ± 0.08	0.13 ± 0.08	−0.01 ± 0.15	−0.01 ± 0.14
Toe off	Males	−0.18 ± 0.09	−0.18 ± 0.09	0.66 ± 0.08	0.66 ± 0.08	−0.07 ± 0.12	−0.06 ± 0.12
	Females	−0.18 ± 0.12	−0.18 ± 0.12	0.60 ± 0.12	0.60 ± 0.12	−0.13 ± 0.13	−0.13 ± 0.13

Event	Group	$\dot{\phi}_1^{(i)}$	$\dot{\phi}_1^{(i+1)}$	$\dot{\phi}_4^{(i)}$	$\dot{\phi}_4^{(i+1)}$	$\dot{\phi}_7^{(i)}$	$\dot{\phi}_7^{(i+1)}$
Heel strike	Males	2.40 ± 1.36	2.22 ± 1.32	5.23 ± 1.74	5.22 ± 1.64	0.80 ± 0.78	0.85 ± 0.65
	Females	0.56 ± 1.45	0.62 ± 1.63	2.79 ± 1.01	2.82 ± 1.11	−0.06 ± 0.50	−0.09 ± 0.51
Foot flat	Males	−0.23 ± 1.06	−0.30 ± 1.26	0.69 ± 1.41	0.70 ± 1.42	−0.92 ± 0.64	−0.89 ± 0.63
	Females	1.77 ± 1.04	1.73 ± 1.11	0.98 ± 1.37	0.99 ± 1.20	−1.18 ± 0.63	−1.12 ± 0.67
Heel off	Males	0.47 ± 0.09	0.48 ± 0.11	−0.57 ± 0.48	−0.50 ± 0.50	−1.27 ± 0.37	−1.24 ± 0.35
	Females	0.27 ± 1.04	0.23 ± 0.96	0.09 ± 1.46	0.06 ± 1.33	−1.38 ± 0.75	−1.36 ± 0.72
Toe off	Males	−3.75 ± 1.68	−3.73 ± 1.87	5.96 ± 0.62	5.91 ± 0.66	2.47 ± 0.96	2.51 ± 1.05
	Females	−3.32 ± 0.69	−3.35 ± 0.68	6.12 ± 0.82	6.04 ± 0.80	2.78 ± 1.00	2.73 ± 1.02

problems. In addition, we obtained the normal profiles of phase plane portraits by assuming the steady state occurred at the middle of the walkway (12th step of each pass). Two observations supported this assumption. First, the circles were centered on the diagonal lines, which denoted repeatability; second, they enclosed the accumulation regions, which indicated that they represented dynamic equilibria.

Kinematic data that were experimentally acquired from a controlled group of healthy subjects were utilized to establish the effectiveness of the proposed methods in analyzing nonpathological gait. The gait parameters obtained during the present study compared favorably with the ones given in Whittle (1991) (97–137 steps min^{-1} cadence and 0.94–1.66 m s^{-1} velocity were reported for females of ages between 18 and 49; 91–135 steps min^{-1} cadence and 1.10–1.87 m s^{-1} velocity were reported for males of ages between 18 and 49).

The advantage of using phase plane portraits is the direct correlation of the joint rotations with the respective joint velocities. Perhaps, the same can be achieved by placing a velocity plot and a position plot above each other. Yet, eliminating time from the graphical representation of the state variables of periodic systems provides a better understanding of the steady-state dynamics. The main usefulness of time in an angle–time plot is that it marks various phases of the gait cycle. The same demarcation can be achieved on the phase plane portrait [see Fig. 3(a)]. However, angle–time and velocity–time plots have to be used together to obtain the same information that is in one phase plane portrait. For example, if one attempts to observe the range of variations in velocities and positions during a given interval of the gait cycle, this will be impossible to observe from angle–time plots only. One can also observe that averaged sagittal phase portraits of 11 female and nine male subjects [Figs 3(b) and (c)] do not significantly differ from each other. The most obvious difference was observed in the hip plots, where the ranges of variation both in velocity and position in females were appreciably larger than in males.

The effectiveness of the first return maps in capturing steady-state gait can be demonstrated by considering the dispersion of the iterates in Fig. 4. One can easily identify the iterates that correspond to transient locomotion resulting from the turns at the ends of the walkway, gait initiation, and stoppage as the subject performed his passes. These iterates lie outside the accumulation regions and can be clearly seen in the figures. This identification would have been very difficult from the observation of time plots of the kinematic variables. We should also point out that the joint positions alone cannot identify the equilibrium values because they are only a subset of the state vector. Thus, the velocities are also needed to characterize the equilibria. For example, an individual may walk at two different velocities by performing a set of movements that are identical but faster. In this case, the first return maps that correspond to the positions will be identical but the velocity maps will be different.

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