On the Measurement of Dynamic Stability of Human Locomotion[1]

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The main focus of the present investigation is the development of quantitative measures to assess the dynamic stability of human locomotion. The analytical methodology is based on Floquet theory, which was developed to investigate the stability of nonlinear oscillators. Here the basic approach is modified such that it accommodates the study of the complex dynamics of human locomotion and differences among various individuals. A quantitative stability index has been developed to characterize the ability of humans to maintain steady gait patterns. Floquet multipliers of twenty normal subjects were computed from the kinematic data at Poincaré sections taken at four instants of the gait cycle, namely heel strike, foot flat, heel off, and toe off. Then, an averaged stability index was computed for each subject. Statistical analysis was performed to demonstrate the utility of the stability indices as quantitative measures of dynamic stability of gait for the subject population tested during the present study.

1 Introduction

Gait studies play an important role in diagnosis and treatment of gait problems and in the design and development of medical techniques and devices used to mitigate these problems. Deterioration of the ability of individuals to walk in a repetitive and stable manner is an obvious manifestation of many pathologic gait conditions. The solution of these problems is the main concern of gait assessment studies. Although stable walking is one of the main factors that the clinicians look for during assessment procedure, a dynamic stability analysis of human locomotion has never been addressed before. This study seeks to characterize the stability of normal locomotion by using an approach based on nonlinear theory that can be utilized in gait assessment process.

Several quantitative measures were proposed by various investigators to assess normal and pathological gait. Robinson and Smidt (1981) proposed to use the three *General Gait Parameters* (cadence, stride length, and velocity) to identify normal and abnormal patterns. Energy consumption is another measure that is used by various investigators. Winter et al. (1976) studied the energy levels of limb segments and Winter (1983) calculated the energy generation and absorption at knee and ankle for different walking speeds. Some investigators proposed to use the heart beats recording rather than energy consumption since it is easier to monitor heart rate than measuring the amount of oxygen uptake. Steven et al. (1983) suggested a "physiological cost index" based on the heart rate measurements. Winter (1987) used the electromyographic and biomechanical analyses to address the sagittal plane balance and posture. He stated that the dynamics of the control of posture and balance of the trunk is completely different than static balance and such a comparison is meaningless. Simon et al. (1983) used external torques along with other kinematic and kinetic measures to characterize the functional results of total knee arthoplasty in a group of twelve elderly patients with isolated degenerative arthritis of one knee. Prodromos et al. (1985) used knee adduction moments as quantitative measures to evaluate the gait of patients with high tibial osteotomy.

The basic physics that govern the dynamic behavior of bipedal locomotion have also been investigated previously. Inverted pendulum models of various complexities have been extensively used in modeling of bipedal gait of humans and walking machines. Apkarian et al. described a model to represent the kinematics and dynamics of the lower limb (1989). Pandy and Berme presented a numerical model for simulating the movement of the lower extremities. Their method offers a compact alternative to manually deriving the equations required for mathematical modeling of human gait (1988). Amirouche et al. (1990) emphasized the importance of the constraint equations in the simulation of the human locomotion for a particular model. Katoh and Mori (1984) proposed a control method to regulate the gait of a simple biped. A dynamical system having a stable limit cycle was used as a reference model to develop the control algorithm. Hurmuzlu and Moskowitz (1987) proposed a new approach to study the dynamic stability of walking machines. Hurmuzlu (1992) studied the global bifurcations of a five element planar bipedal model. He also investigated the steady-state behavior and stability of the motion by using numerical techniques. This common effort culminated in important results in terms of developing methodologies to analyze gait and answering some fundamental questions regarding its dynamics and control. However, the focus of these investigations was the study of man-made bipedal systems with well defined internal components and motion strategies. Yet, when the system is human, the structure

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is extremely complicated. There is insufficient information available about the control strategies that are used by human beings to achieve stable gait.

Despite the extensive effort in the area, there is no accepted quantitative way to judge or score the dynamic stability of human locomotion. The process of determining the stability of particular patient still hinges on the personal observations and past experience of the involved team members. In the present study we propose a new approach that can be used to study the dynamics of the periodic motions that arise during human locomotion. Instead of the commonly used single step analyses, we focus on the investigation of the dynamic stability of the overall gait process. Application of the proposed methods does not require any assumptions regarding the inner structure of the human system nor the strategies that it utilizes in actuating its motion. We simply collect the displacement data of the lower extremities and capture the instants of gait cycle to conduct the analysis.

In this article, we first outline the fundamental aspects of the theory that is used in the present investigation. Then, we develop stability measures to quantify the robustness of periodic gait patterns subject to internal and external perturbations. Finally, statistical techniques have been used to describe the results meaningfully and to compare the stability measures of individual subjects at specific sections of the gait cycle, namely heel strike, foot flat, heel off and toe off.

2 Theoretical Background

The development presented here is based on the Floquet theory, which is used to investigate the local stability of critical points of discrete maps. In simple terms, we would like to quantify the resilience of the periodic gait patterns when subjected to disturbances. We are particularly interested assessing the ability of a human biped to establish regular gait while walking with normal speed.

2.1 Nonlinear Dynamical Systems and Phase Plane Portraits. Mathematical analysis of a nonlinear systems such as bipeds requires the identification of a set of variables that depict the underlying dynamics (state variables). Then graphical and analytical techniques can be used to further investigate the bipedal motion. Here we will present a simplified model to present the basic approach that will be applied to the study of human motion. The three-link planar bipedal model that is depicted in Fig. 1(*a*) is used. We further simplify the problem by neglecting the dynamics of the feet, assuming purely rotational joints, and nondeformable members. The motion of this system evolves in the six-dimensional state space, $\{\phi_1, \phi_2, \phi_3, \dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3\}$.[2] We will use the $\phi_1 - \dot{\phi}_1$ projection of this space to depict the trajectories that correspond to the locomotion of the bipedal model.

Figure 1(*b*) depicts the evolution of one step of locomotion of the biped on the phase plane portrait of the generalized coordinate ϕ_1. Point 1 on the phase plane portrait is the time instant when leg A is the stance limb and leg B is the swing limb and the biped is swinging in the forward direction. At point 2, the swing limb contacts the ground surface and as a result of the impact with surface we observe a sudden change in the velocity (Point 3). When the contact occurs, the pivot point transfers to the contact point of limb B with the ground. Following the contact event the biped continues to its forward motion now swinging on leg B. This motion continues until the second contact occurs at Point 4, and the cycle of events repeat thereafter.

The motion that is depicted in Fig. 1(*b*) is not periodic because points 1 and 5 of the phase trajectory do not coincide. Yet, for bipedal models of this type one can identify a family

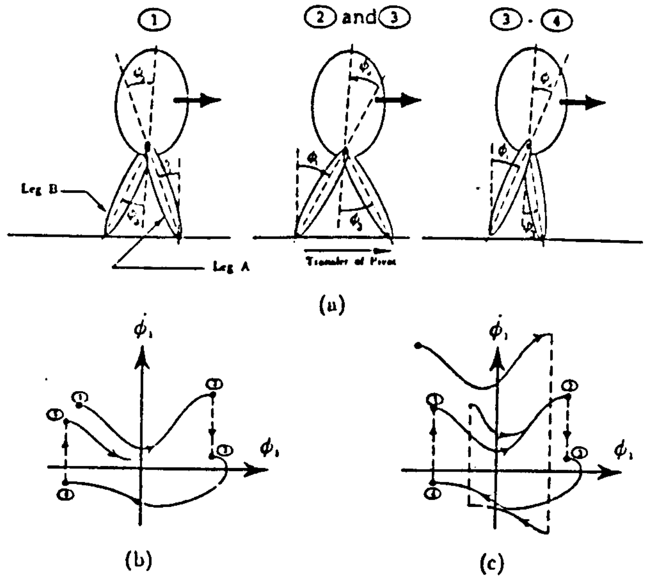

Fig. 1 Phase plane portrait and periodic motions of a three-link model

of periodic motions with phase plane portraits that are similar to the one depicted in Fig. 1(*c*) (Hurmuzlu, 1992). When the motion becomes periodic, the trajectory loop closes on itself as shown in the figure (loop 1-2-3-4). In general, one expects to observe several transient locomotion steps before the system attains periodicity. Evolution of this type motion is depicted by the trajectory marked in Fig. 1(*c*).

One can easily represent the evolution human gait by using projections of the phase space on the phase plane portrait of a particular joint displacement. This requires the knowledge of the angular displacements and velocities at the joints that are being considered. Angular position data can be obtained experimentally and the velocity information can be computed by numerically differentiating the position data. For the human biped, however, one should not expect to see the sudden changes in velocity that are observed at the contact points of the rigid body model. Effect of heel strike in human locomotion is considerably less critical because the impulsive forces are absorbed by the soft tissue.

2.2 Periodic Motions and the Poincaré Map. A classical technique to analyze dynamical systems has been developed by the nineteenth century French mathematician Henry Poincaré. A formal mathematical description of the technique can be found in many previously published books and articles (see Guckenheimer and Holmes, 1985 and Parker and Chua, 1989). Here we follow an informal presentation that describes the main features of the approach, its advantages, and its relevance in gait studies.

In the three-link bipedal model of the previous section, we have shown that periodic motions of the system are represented by closed orbits in the phase space. Figure 2(*a*) depicts a hypothetical case where a particular trajectory descends on the closed orbit with successive locomotion steps. The points of the trajectory that coincide with the instant of heel strike are labeled as p_i and the point of the closed orbit at heel strike is labeled as p_e. Poincaré map for the generalized coordinate ϕ_1 at the instant of heel strike now can be obtained by plotting the values of ϕ_1 at p_i versus the values at p_{i+1}. This construction is depicted in Fig. 2(*b*). The iteration point that corresponds to the closed orbit (p_e) is on the 45 deg line because when the motion is periodic ϕ_1 at heel strike is identical for all successive locomotion steps. We also observe that points on the map accumulate at p_e as the biped takes successive locomotion steps. The same construction can also be performed for the gener-

[2] Dots denote differentiation with respect to time. For example $\dot{\phi}_1$ denotes the angular velocity at joint 1.

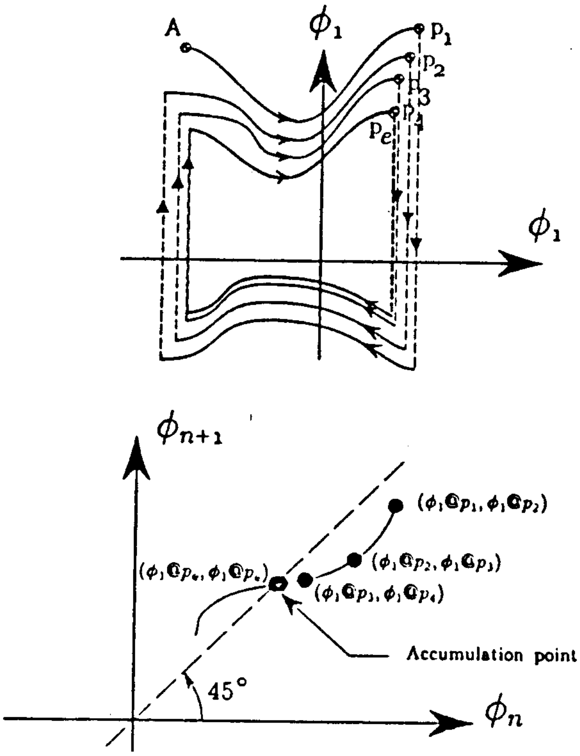

Fig. 2 Equilibrium points and the first return map

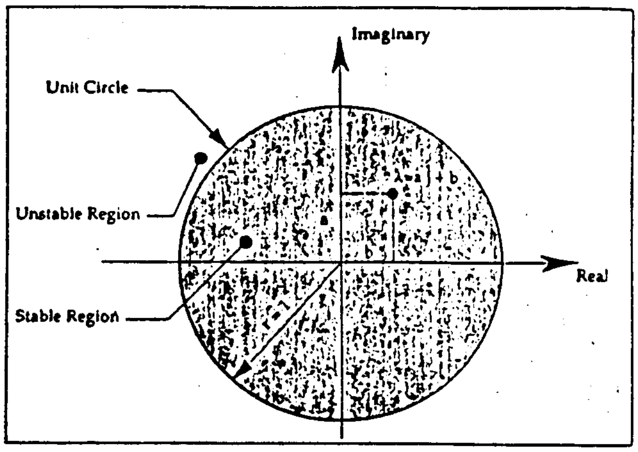

Fig. 3 Stable and unstable regions on the complex plane

alized velocity $\dot{\phi}_1$ or for any other kinematic quantity. One can equally construct a discrete map for the velocity of the center of mass of the upper member and investigate the evolution of locomotion speed with successive steps. Instead of choosing heel strike we could have chosen the instant of toe off for the present model and obtained similar plots.

2.3 Stability of Periodic Motions and the Floquet Multipliers. Now we present an approach to quantify the stability of the periodic motions that are discussed in the previous sections. The development is based on Floquet theory, which is used to investigate the local stability of critical points of discrete maps.

For the simplified bipedal system that we have considered above, the state variables are the three generalized coordinates (ϕ_1, ϕ_2, and ϕ_3) and three generalized velocities ($\dot{\phi}_1$, $\dot{\phi}_2$, and $\dot{\phi}_3$). Then, the state vector **x** is given by

$$\mathbf{x} = [\phi_1, \phi_2, \phi_3, \dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3]^T \quad (1)$$

Accordingly, the analytical representation of the graphical construction that has been described in the previous section can be written as

$$\mathbf{x}_{i+1} = \mathbf{f}^{HS}(\mathbf{x}_i) \quad (2)$$

where, $\mathbf{x}_i$ and $\mathbf{x}_{i+1}$ represent the state vector at the Poincaré section during the ith and $(i+1)$th steps, respectively. The function $\mathbf{f}^{HS}$ is the discrete map that represents the dynamics of the model. The superscript *HS* on the function denotes that the Poincaré section is taken at the instant of heel strike. As stated earlier, other instants of the gait cycle can also be used to construct the discrete map, and the function can be labeled accordingly.

Since this is a man made system, one can obtain the analytical representation of the function $\mathbf{f}^{HS}$. Periodic motions of the system correspond to the critical points of this map. If we consider the point p_e that is depicted in Fig. 2, we can write,

$$\mathbf{x}^e = \mathbf{f}^{HS}(\mathbf{x}^e) \quad (3)$$

where the subscript on the state vector denotes that the state corresponds to the heel strike instant when the motion of the biped is periodic. Linearizing the map about the equilibrium point $\mathbf{x}^e$ gives,

$$\mathbf{x}_{i+1} = \mathbf{J}^{HS}(\mathbf{x}_i) \quad (4)$$

where $\mathbf{J}^{HS}$ is the Jacobian matrix. The matrix $\mathbf{J}^{HS}$ is an 6×6 constant coefficient matrix. The eigenvalues of this matrix are called the Floquet multipliers and can be used to determine the dynamic stability of the periodic motions of the system. The theory states that the motion is stable when the magnitudes of all Floquet multipliers are less than unity. We should point out that the eigenvalues can be real or complex numbers. Alternatively, the condition for stability can be represented graphically as shown in Fig. 3. The figure depicts the unit circle on the complex plane (i.e., the horizontal axis represents the real part of a complex number and the vertical part represents its imaginary part). Then, the system is stable when all the Floquet multipliers fall inside the darkly shaded region shown in the figure.

Analytical derivation of a discrete mapping that reliably reflects the dynamics of the human locomotion is not a trivial task because of the complexity of the human body. Yet, one can obtain the Jacobian matrix of the linearized map by experimentally measuring the data and using curve fitting techniques. This procedure requires a fundamental assumption to be made regarding the number of segments that form the human body. This is required because the theory is based on the state variables of the system being considered. In this study we have assumed that the human body is composed of seven segments. These include the feet, shanks, thighs and the upper body (HAT). In the study of normals, we have further simplified our analysis by assuming left-right symmetry. Symmetry of normal gait is an assumption that has been commonly accepted by investigators in the gait analysis field (Inman et al., 1981). We measured nine rotations at the ankle, knee, and the hip of the dominant side. The resulting eighteenth dimensional state vector can be written as

$$\mathbf{x} = [\phi_1, \ldots \phi_9, \dot{\phi}_1, \ldots \dot{\phi}_9]^T$$

where, the angular positions ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5, ϕ_6, and ϕ_7, ϕ_8, ϕ_9 correspond to the extension-flexion, abduction-adduction, and internal-external rotations at the ankle, knee, and hip respectively.

Nonlinear dynamical theory dictates that the Floquet multipliers do not depend on the choice of Poincaré section. Hence, for the locomotion case, one should be able to compute the same multipliers whether the section is taken at heel strike, foot flat or at any well defined instant during the gait cycle. This would also be true for human locomotion if we would have used a dynamical model that captures all the state vari-

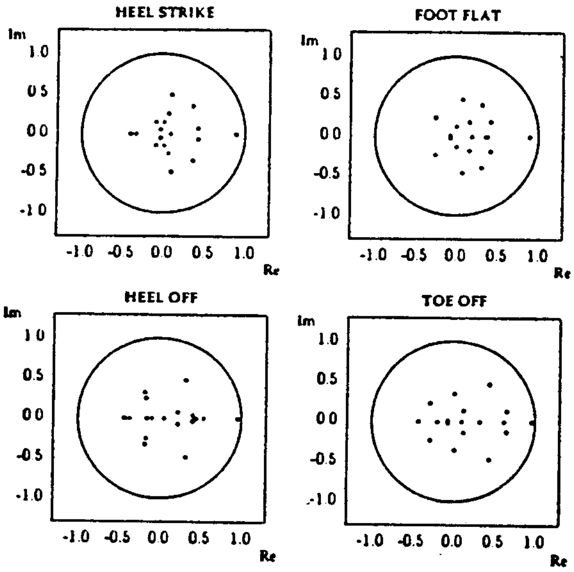

Fig. 4 Distribution of Floquet multipliers on the complex plane for a female subject

ables of the human body. However, the seven segment structure for the human body does represent a highly simplified model. Realistically, we cannot expect the invariance property to hold exactly in our analysis. Nevertheless, the simplification can be justified by studying the difference in the Floquet multipliers resulting from the choice of different sections. Yet, even the basis to perform this comparison is not obvious. This is mainly due to the difficulties associated with tracking the motion of individual multipliers on the complex plane when they undergo discrete changes as a result of changing the Poincaré section. Consider the plots in Fig. 4 that depict the Floquet multipliers of a female subject obtained at the four sections of the gait cycle. There are 18 multipliers in each plot, which were obtained by computing the eigenvalues of the linearized map at various sections. We cannot isolate four points on the four plots of Fig. 4 that correspond to the same Floquet multiplier.

An alternative approach that is proposed here is to define an overall stability measure that reflects the collective behavior of the Floquet multipliers but eliminates the need for tracking the individual eigenvalues. Accordingly we let,

$$\alpha = \frac{1}{2n} \sum_{i=1}^{2n} |\lambda_i| \qquad (5)$$

where n is the number of generalized coordinates that represent the dynamics of the human body and λ_i are the Floquet multipliers computed at a specific instant of the gait cycle. In the present article we define four α-measures, which can be enumerated as follows:

1. α^{HS}—computed for a Poincaré section taken at heel strike
2. α^{FF}—computed for a Poincaré section taken at foot flat
3. α^{HO}—computed for a Poincaré section taken at heel off
4. α^{TO}—computed for a Poincaré section taken at toe off

As we have stated earlier, the stability measures reflect the collective behavior of the Floquet multipliers. Considering the basic properties of the Floquet multipliers the following propositions can be made:

I. The α-measures computed for the normal human subjects at a given section should be similar.
II. The α-measures computed for an individual subject at the four sections of the gait cycle should be similar.
III. Smaller values of α-measures denote a gait pattern that is more resilient to perturbations.

We should emphasize that there are two separate issues that have to be addressed in this investigation. First, we are proposing a new measure to penetrate the dynamic stability of human gait. Second, a simplified human body model was used to demonstrate the application of the proposed approach to gait analysis. Now, the validity of the assumed model and applicability of Floquet theory to the study of human gait can be verified by carrying out statistical analyses that test the first two of the three propositions. The third proposition is a direct extension of the Floquet theory and can be used to identify gait patterns that are different than normal. The statistical analyses and their results are presented later in the article.

3 Experimental Procedure and Data Processing

A modified triaxial electrogoniometer system (Chattex Corp.) was used for data collection in the present study. Joint excursions in sagittal, transverse, and frontal planes at the hip, knee, and the ankle joints of the subject's dominant sides were measured. Foot switches were designed to capture the beginning and end of various events during the gait cycle. Several adjustments to the system, such as cabling and wiring, were considered in order to direct the path of subjects and supply the analog data to the computer.

A twenty-two meter walkway was used to acquire the kinematic data in a continuous fashion. The data were collected from twenty adult subjects (9 males and 11 females), free of pathological problems. The male subjects ranged in ages from 19-36 years (mean = 26.6 ± 7.0), and weighed between 50-85 kg (mean = 68.2 ± 13.3). The female subjects were between 19-49 years of age (mean = 28.9 ± 10.1) and weighed between 50-83 kg (mean = 59.0 ± 9.0). The subjects were requested to perform sixteen successive passes over the twenty meter walkway adopting their most comfortable cadence. The triaxial electrogoniometers described above were attached to the dominant limb of each subject using four nylon straps wrapped around the limb area of the lower-calf, mid-calf, lower thigh, and upper thigh. Potentiometer outputs were fed to a personal computer through A/D converters with a sampling frequency of 100 Hz from each channel. Foot switches were attached to heel and toe of the subject's dominant foot, and then connected to the foot-switch interface unit. Moreover, heart rate of the subjects was monitored using an ECG telemetry system (Transkinetics Inc.). A photo cell system was used to record the average velocity of the each completed pass. The data were stored in a complete report form for further processing.

The raw data acquired in the gait laboratory were stored in separate files corresponding to each individual subject. Each file included the records of the nine rotational joint rotations and the foot switch information. Subsequently, displacement data were smoothed by using a second degree smoothing procedure to eliminate the amplification errors and reduce the sensitivity of velocity computations to noise. Joint velocities were computed by using a numerical differentiation technique. The state vector **x** was reconstructed by using the measured joint rotations and the computed joint velocities. The α-measures for each subject were computed by carrying out the following steps:

1. Using foot switch information kinematic data were extracted at the four sections of the gait cycle for the sixteen passes performed by each subject. The resulting data were arranged in the form of four matrices that were given by,

$$\mathbf{A}^{HA}, \mathbf{A}^{FF}, \mathbf{A}^{HO}, \text{ and } \mathbf{A}^{TO}$$

where superscripts denote the respective sections. Each array had a dimension of $m \times 18$, where m was the number of steps taken by a particular subject during the 16 passes. The rows of each array were formed by using the suc-

Table 1 Descriptive statistics at four section of step cycle for males ($n=9$) and females ($n=11$)

α-measure	Sex	Mean	S.D.
α^{NC} (No Contact)	Male	0.395	±0.050
	Female	0.391	±0.053
α^{HS} (Heel strike)	Male	0.357	±0.042
	Female	0.378	±0.061
α^{FF} (Foot flat)	Male	0.343	±0.026
	Female	0.337	±0.045
α^{HO} (Heel off)	Male	0.349	±0.039
	Female	0.377	±0.062

cessive values of the state vector $\mathbf{x}$ at the respective sections.

2. Four $m-1\times 18$ dimensional, first return matrices given by

$$\mathbf{B}^{HS},\ \mathbf{B}^{FF},\ \mathbf{B}^{HO},\ \text{and}\ \mathbf{B}^{TO}$$

were defined. The components of the first return matrices were computed from,

$$\mathbf{B}_{i,j}=\mathbf{A}_{i+1,j}\quad i=1,\ m-1\quad\text{and}\quad j=1,\ 18 \tag{6}$$

where i and j denote column and row numbers, respectively. Here, the nth rows of the matrices $\mathbf{A}$ and $\mathbf{B}$ represent the vectors $\mathbf{x}_n$ and $\mathbf{x}_{n+1}$, respectively.

3. The components of the Jacobian matrix, $\mathbf{J}_{j,k}$ and the constant vector Γ_j were computed from 18 linear fits of the form

$$x_{i+1}^{j}=\sum_{k=1}^{18}\mathbf{J}_{j,k}x_i^{j}+\Gamma_j\quad\text{with}\quad j=1,\ 18 \tag{7}$$

by using the experimental data stored in the matrices $\mathbf{A}$ and $\mathbf{B}$ and a least squares algorithm.

4. Floquet multipliers were found by computing the eigenvalues of the 18×18 matrix $\mathbf{J}$.

5. Finally, the α-measures were determined by using the eigenvalues found in the previous step in Eq. (5).

3.1 Descriptive Statistics. Mean and standard deviation were used as basic statistical quantities to characterize the distribution of the magnitudes of the Floquet multipliers. This identification was important to establish the statistical properties of the α-measures of the normal population.

Results of descriptive statistics at four sections of step cycle for the 20 normal subjects tested in the present study are presented in Table 1. We observe that the means of the α-measures are fairly similar for male and female subjects. Scatter of the α-measures around the means vary in the range 7.6–15.7 percent standard deviation. Figure 5 depicts the α-measures calculated for all subjects at the four sections. For stable locomotion, the measures are expected to fall into a range that is bounded by the values of 0 and 1. Yet the variation range of the actual measures for normal subjects was found to vary between the values of 0.27–0.5.

3.2 Inferential Statistics. The goal of this section is to verify the first two propositions which were described in the theory section of this paper. In this regard, two-way analysis of variance (ANOVA) was performed to explore the invariance properties of stability measures across the subject population (Proposition I) and across the sections (Proposition II). For this purpose, four sets of eighteen multipliers were computed for each of the twenty subjects in the study group. These sets were obtained by using Poincaré sections taken at the four instants of the gait cycle and applying the procedures described in the previous section. The null hypotheses that were used in ANOVA were formulated as

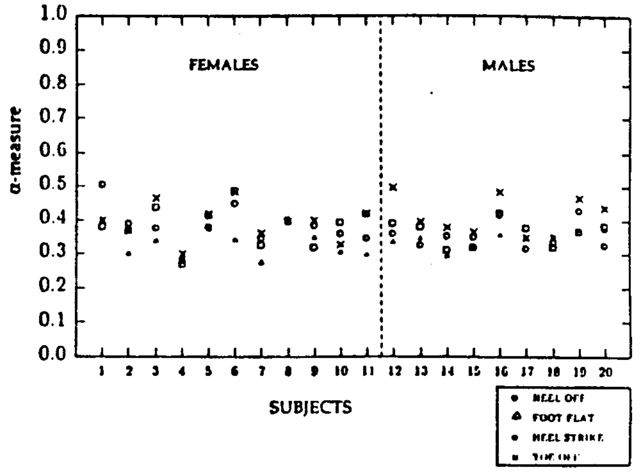

Fig. 5 Variation of stability indices at the four sections of the gait cycle for females ($n=11$) and males ($n=9$)

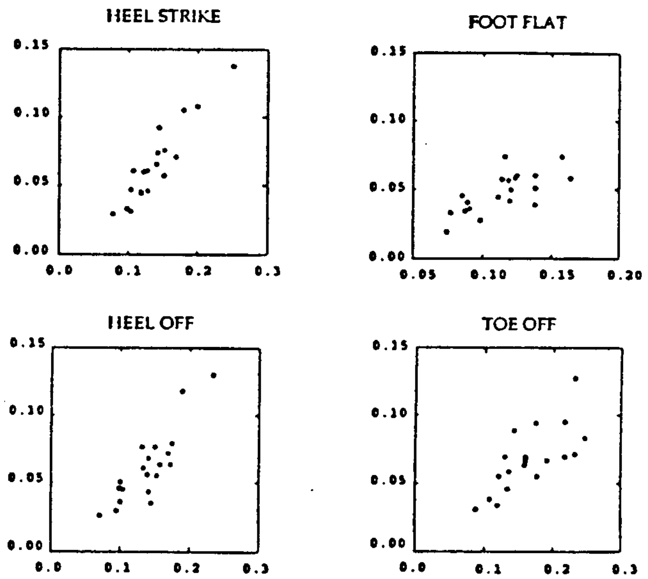

Fig. 6 Variances versus squares of means at four sections for 20 subjects

$$\mathrm{H}_0\text{: }\alpha_1=\alpha_2=\cdots=\alpha_{20} \tag{8}$$

$$\mathrm{H}_0\text{: }\alpha^{HS}=\alpha^{FF}=\alpha^{HO}=\alpha^{TO} \tag{9}$$

where the former null hypothesis corresponds to proposition I and the latter to proposition II. The subscripts and the superscripts on the α-measures given in the equations denote the subjects and the sections of gait cycle respectively. In the course of the analysis the subjects were treated as the grouping variables and the multipliers were treated as the continuous variables.

First, we conducted a set of analyses to justify the application of ANOVA to the study of the Floquet multipliers. Application of ANOVA requires homogeneous group variances and normally distributed data. The group variances computed from the original values of the multipliers were not homogeneous. As a result, the data were transformed by using a simple logarithmic transformation to obtain homogeneous group variances. This transformation is appropriate when the variances are approximately proportional to the square of the means. Figure 6 depicts the plots of variances versus squares of means of multipliers obtained for the 20 normal subjects at the four instances of the gait cycle. Based on the noticeably linear profiles of these plots we concluded that the transformation was

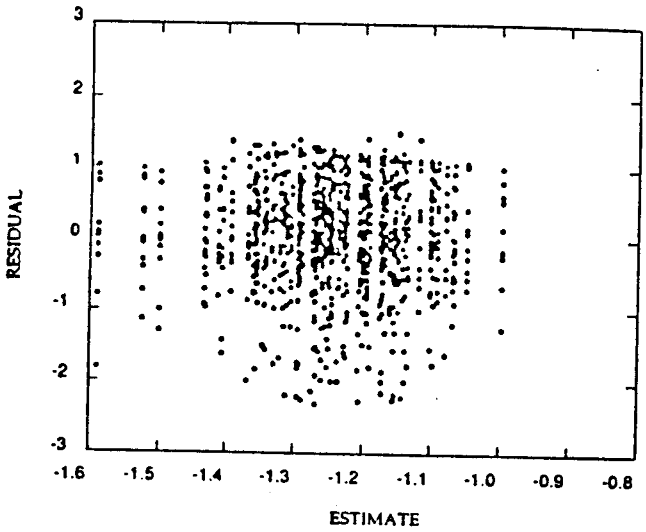

Fig. 7 Residuals versus estimated values of multipliers

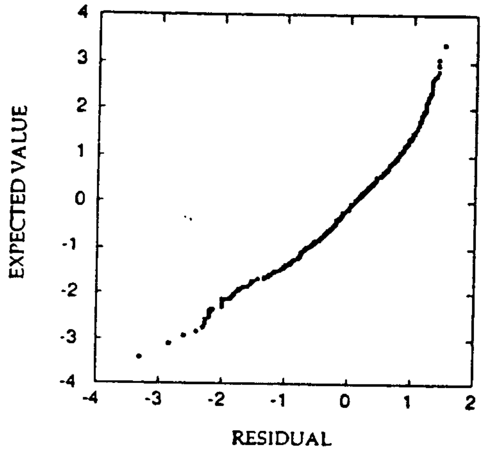

Fig. 8(*a*) Probability plot of residuals

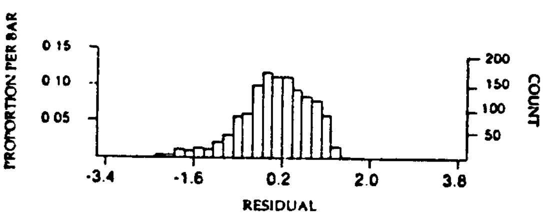

Fig. 8(*b*) Histogram of residuals

warranted. Homogeneity of the variances of the transformed data was checked by plotting the residuals against the estimated values of multipliers. The random scatter of the points within a band around the horizontal axis of the plot verified the validity of the transformation (see Fig. 7).

The normality assumption was verified by plotting the probabilities of the residuals (Fig. 8(*a*)). Although the points in the plot did not exactly lie on the diagonal, as would have been observed for perfectly normal distributions, we did not detect extreme departures from normality. Additional evidence of normality was found when the histogram of residuals was considered. We observe that the histogram did not show significant deviations from the standard normal distribution curve (see Fig. 8(*b*)).

Having demonstrated that the data satisfies the normality and homogeneity requirements, we now present ANOVA tables obtained for the current study. The results of two-way

Table 2 Two-way ANOVA table for 20 subjects at four sections

ANOVA Table				
Source	Sum of Squares	DF	F-Ratio	Prob.
Subject	12.823	19	1.052	0.397
Section	4.626	3	2.403	0.066

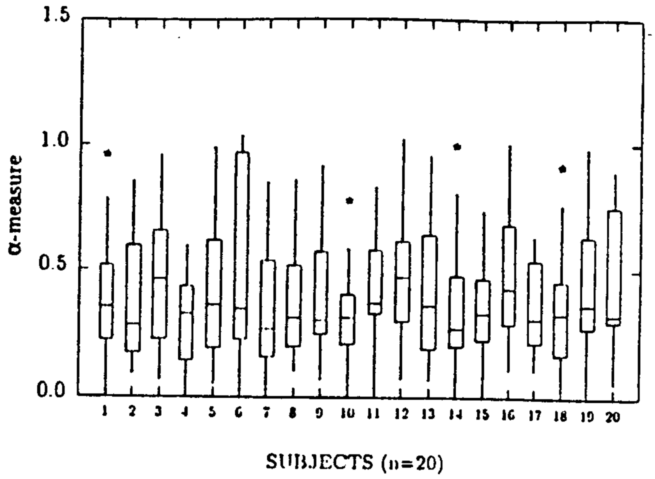

Fig. 9 Box plot of stability indices at toe off for 20 subjects

ANOVA for α-measures among 20 normal subjects at four sections are tabulated in Table 2. As seen from the probability values listed in the last column of Table 2, results of ANOVA strongly suggested that there was no basis for rejecting the first null hypothesis (i.e., statistical means of Floquet multipliers or more specifically α-measures, did not vary across the tested subject population). The probability value computed for proposition II was higher than the commonly used significance level of 0.05. Yet, we could not strongly conclude that none of the α-measures of a given subject were significantly different than each other at the sections.

Finally, Fig. 9 depicts the box plots of the Floquet multipliers computed at toe off for subject group. The figure provides a simple graphical summary of the distribution of Floquet multipliers around the respective medians. The general trend in individual boxes demonstrated that there was a concentration of multipliers on the lower side of median. We also observed that the distributions of the multipliers did not differ significantly from one another. A striking aspect of the box plots was the number of multipliers that fell outside the given distributions. In a total of 360 multipliers at each section, at most four multipliers fell outside the distributions, while none fell in the far outside of the given distributions. We observed that the elimination of the outside values did not alter the fundamental conclusions of the analysis of variance.

4 Discussion and Conclusion

A quantitative measure of the stability of human locomotion can provide the clinical team an essential mean to diagnose gait pathologies, administer proper treatments, and monitor patient progress. We have presented an analytical methodology that can be used to score the dynamic stability of bipedal gait. An experimental study was conducted to characterize the stability of self adopted gait patterns of normal individuals. Kinematic data and foot switch information from the dominant sides of 11 female and 9 male subjects were experimentally measured. Then, the proposed analytical techniques and the experimental data were used to compute the stability measures of normal human locomotion.

We have chosen electrogoniometers as the principal measurement device in the data acquisition process. On one hand, there are two disadvantages associated with the usage of these devices. First, fewer number joint measurements can be made compared to more advanced optical data acquisition systems. Second, it is well known that the potentiometers that measure joint rotations cannot be consistently aligned along anatomical joint axes. One the other hand, goniometers are easy to use and widely available in gait laboratories. In addition, once they are in place, they can effectively measure rotations along the three mutually perpendicular axes of the lower extremity joints. As far as the present investigation is concerned, the first disadvantage has necessitated a simplifying assumption to be made regarding the number of segments that represent the human body. It was assumed that the human body is composed of seven segments (one torso and six segments for the lower extremities). Yet, we have demonstrated that this assumption did not impair the outcome of the stability analysis. Results of the statistical analysis has demonstrated that the experimentally computed stability measures do not show significant variations among subjects. We have further shown that the stability measures are fairly insensitive to the choice of Poincaré section. Thereby, we have established that a fundamental aspect of the underlying theory is preserved in the experimentally computed measures. The second disadvantage associated with the usage of electrogoniometers is eliminated by the proposed methodology. Accurate prediction of Floquet multipliers depends on the number of degrees of freedoms used to represent a particular joint. The potentiometers measure the rotations along three orthogonal axes. Hence, to a great extent they capture the three degrees of freedom associated with that joint (neglecting translational motions that may occur in the knee joints). Therefore, the method is not affected by the misalignment of the potentiometers with the respective anatomical axes. The misalignment problem is not exclusive to the use of electrogoniometry. Such problems may also arise in optical systems where positioning of markers does not necessarily ensure the exact allocation of the rotation axes of the anatomical joints. Thus, insensitivity to deviations from anatomical axes is an important advantage of the proposed method.

An important feature of α-measures is that they penetrate a fundamental aspect of human gait. One of the most obvious signs of normal locomotion should be the robustness of periodic gait patterns when subjected to disturbances. We have discovered that normal individuals possess stability measures that are substantially less than unity. This result is important, because it conforms with the theory and fulfills our intuition regarding the stability of normal gait. An important issue, however, was not addressed in the present study. The usefulness of these measures in clinical gait studies impinges on their sensitivity to pathological gait conditions. Future studies testing hypothesis that pathological gait leads to higher stability measures are required to establish the utility of the proposed methods in clinical gait analysis. Such studies should include patient populations whose pathological problems are difficult to characterize using single step analysis such as Post-Polio patients. Users of lower limb prostheses is another population that may potentially benefit from such investigations. Presently, there is little agreement both on the techniques and the methods used in the selection and evaluation of prosthetic devices. Although alignment procedures are ultimately effective, they are lengthy and vary significantly with the prosthetist performing the alignment. The proposed stability measure may greatly facilitate the selection and alignment process.

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