MECH 534 Computer-Based Modeling and Simulation
Notes on *Decision Trees and Monte Carlo Simulations*

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<u>Decision Trees</u>: Decision trees are typically used to support decision-making in an uncertain environment. For example, in making engineering decisions for product manufacturing, the engineer usually faces multiple unknowns that make it difficult to choose a winning option. Although the engineer does not know what the overall outcome will be, he generally has some knowledge—or at least an opinion—about what the possible outcomes for the various phases of the operation and how likely each is to occur. This information can be compiled to help the option that is most likely to yield favorable results. Decision trees make this type of analysis relatively easy to apply.

A decision tree has three types of nodes: (a) decision node (b) chance node, and (c) leaf node. The branches originating from a decision node represent options available; those originating from a chance node represent uncontrollable events. At each chance node, each branch is assigned a conditional probability equal to the probability of the event represented by the branch, conditioned upon the knowledge available at the node. Leaf nodes represent the possible endpoints, i.e. the results of the decisions and chance outcomes associated with the path from the start of the tree (also known as the *root*).

<u>Decision Tree Analysis:</u> If you could somehow determine precisely what would happen as a result of choosing each option in a decision, making decisions would be easy. You could simply calculate the value of each competing option and select the one with the highest value. In engineering decisions, where there is a considerable amount of uncertainty and where the possible outcomes are quite complex, decisions are not that easily made. The objective of finding the optimal solution—that is, the best set of choices at the decision nodes—can be achieved by applying a "roll-up" process to the decision tree. Starting with the leaf nodes and progressing recursively toward the root, we label each node by the value of the situation it represents. Each chance node is labeled with the expected value of its successors, and each decision node is labeled with the value of the choice that has the largest value.

Consider the following example to describe the "roll-up" concept.

Suppose you are an engineer working in the production line and you are asked to manufacture a product as fast as possible so that the company can enter the consumer market earlier than the competitors. Now, let's assume that there are two different manufacturing methods (see Figure 1). One (Method A) is possibly quicker (if everything goes smooth, the production time is 30 minutes) but there is a chance that 30 % of the products manufactured using this method may come defective from the production line (note that the defective parts have to be manufactured again which results in a longer production time at the end: 60 minutes). The production time for the other manufacturing method (Method B) is longer (50 minutes) but the process does not create any defect on the parts. Which one should you choose?

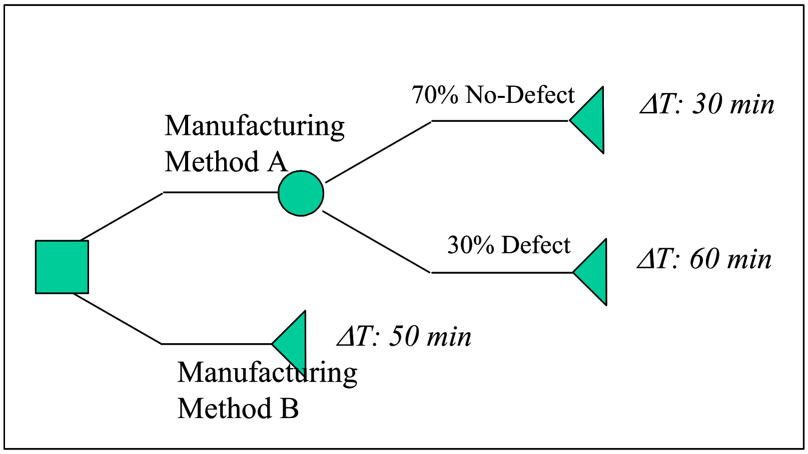

Figure 1. An example case to describe the "roll-up" concept. The decision nodes are represented by squares, chance nodes by circles, and leaf nodes by triangles.

The roll-up approach discussed earlier can be used to choose the best option. In roll-up analysis, the expected value at a chance node can be calculated by multiplying values along the branches by its probability and adding the results together.

$$\text{Expected Value} = \sum_{i=1}^{N} \text{Value}_n \times \text{Pr obability}_n$$

On the other hand, the expected value at the decision node is that of the best option (e.g. minimum time, maximum strength, etc.)

Based on these rules, the Expected Production Time (EPT) associated with the Method A is 39 min (30*0.7+60*0.3) in our sample case. Since the EPT for the longer operation is 50 minutes, we should choose the fast operation as the best option (i.e. MIN(39,50)) if our criteria for the success is purely based on the production time. However, it quickly becomes apparent that our expenses will increase if we choose the faster manufacturing operation since we have to inspect parts to search for defects. Perhaps, we will produce the parts on time and enter the market early, but the cost of this operation will be so high that we will not be able to make sufficient profit from the product in the long run.

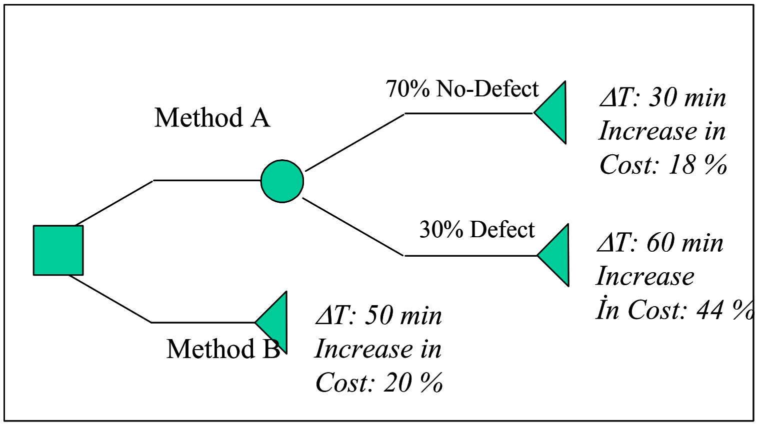

Figure 2. An extended version of the sample case depicted in Figure 1.

Let's continue to study our problem. Let's assume that increase in production cost is given for each branch (see Figure 2). Now, the Expected Increase in Cost (EIC) for fast manufacturing method (Method A) will be equal to 26% (18*0.7 + 44*0.3). Since the EIC for the slow, but less defective manufacturing method (Method B) is equal to 20% (see Figure 2), we should choose (i.e. MIN (26,20)) the slow manufacturing method if our main concern is the cost of production. As can be seen in these examples, there are always tradeoffs in decision making. Since the criterion for the success is "manufacturing the product as fast as possible with minimum cost", we need to consider both variables (i.e. production time and cost) to make a decision at the highest level of the tree.

This analysis assumes that everything important has been included in the values assigned to the leaf nodes. However, it is usually not easy to estimate the exact values of variables along the branches. For example, in our case, we assumed that production time for Method B would be 50 minutes. However, in reality, we know that this will very unlikely to take *exactly* 50 minutes, so we might want to conduct sensitivity analyses later, in which we vary the value over the entire range of likely values. In fact, by using a Monte Carlo approach, as discussed in the next section, we can do the sensitivity analyses when we process a decision tree. Production time, for example, might be described by a 40 minute *minimum* value, a 50 minute *most likely* value, and a 70 minute *maximum* value.

Monte Carlo Simulations:

Decision trees, which are discussed above, provide an excellent tool for analyzing the consequences of alternative decisions for selecting contingent actions to take in response to events. The so-called *Monte Carlo* approach provides a convenient means to consider all of the uncertainties in a decision tree. Instead of applying a single number to each input variable (e.g., time, attrition, etc.) and receiving a single number back from the calculation engine, a probability distribution is applied to each input variable. This probability distribution represents the uncertainty in the input variable. A random value is then drawn from each probability distribution and the output value measures are calculated. By applying this procedure repeatedly (perhaps hundreds or thousands of times) and plotting a histogram of the output value measures, a risk profile for the project is built.

In traditional analyses of decision trees, the only probabilities considered are those that determine which branch is taken at chance nodes. One consequence is that only discrete possibilities can be considered (For example, the production time in our example above could only take one of the three values 35 minutes, 50 minutes, or 60 minutes). In the Monte Carlo approach, we describe the production time by a probability distribution and then randomly select a value from that distribution. This operation is repeated many times, drawing new random numbers for each uncertain parameter each time. Providing the appropriate statistics are collected to describe the outcomes of many trials, it then becomes possible to make decisions on the basis of the probabilities associated with the entire range of results. That is, sensitivity analysis is built into the results.

Monte Carlo iteration facilitates statistical analysis of problems that are not otherwise easily solvable. The decision trees associated with engineering decisions are often of this nature as there are many complications, like occurrence of defects that make it difficult to assess the effect of a change in an input variable to output without redoing the entire calculation from the start. Especially, when there are several uncertain values, determining the uncertainty of the result can be very complex. Since multiple actions with identical expected values can have completely different risk profiles, a single number that represents the best estimate for the value of an opportunity is often insufficient, as the uncertainty in this estimate must also be evaluated. Monte Carlo iteration is a convenient and accurate method of doing this.

Choosing a Probability Distribution: As we discussed above, one thing the Monte Carlo approach allows us to do is to estimate the probability distribution of input variables, rather than having to rely on solely the most likely value. This allows the decision maker to express his uncertainty directly, rather than having to specify single values. In our study of Monte Carlo simulations, the user will input some of the numbers, then associated simulations will generate other values and their uncertainty distributions along the branches of the decision tree. We will use triangular probability distribution functions to specify min, most likely, and max values, entered directly by the user (see Figure 3).

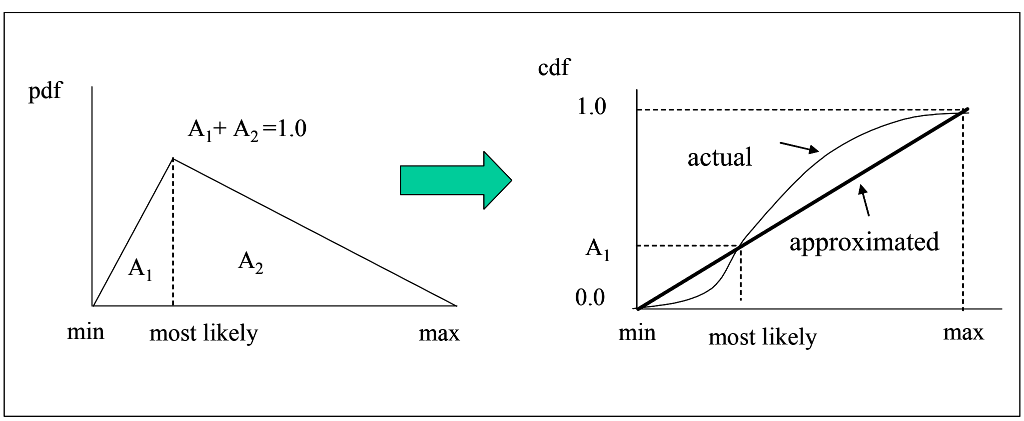

Figure 3. A triangular pdf and the cdf. The user specifies the min, most likely, and maximum values.